

Do Copula-Extreme Value Theory-based
Semiparametric Approaches Perform Better when
using the Value-at-Risk Measure?

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Abstract

Measuring Value-at-Risk (VaR) is an important function in financial risk management. One of the most popular methods of computing VaR is the Monte Carlo simulation, which focuses on utilizing an appropriate approach to estimate the dependence between returns of financial assets. However, the existence of fat-tailed, skewed distributions and non-linear relationships of financial asset returns makes conventional Pearson product-moment coefficient approach incongruous. To overcome this difficulty, the current research applies the extreme value theory (EVT) in order to model the tails of the return distributions and copula functions to build the joint distribution of returns. More specifically, in the copula-EVT-based methodologies, the marginal distributions of asset returns are modeled using a semiparameter approach in which the distribution center is modeled by a nonparameter empirical distribution and the distribution tails are modeled by the generalized Pareto distribution (GPD) with parameters; furthermore, three copula functions—Gaussian, Gumbel, and Clayton—are applied to model the general, upper-tail, and lower-tail dependencies.

To test the advantages of these approaches, six Asian countries were selected based on their different stock index and foreign exchange return distribution shapes, and backtestings were conducted to examine the Monte Carlo VaRs simulated from the correlation coefficients estimated by the Pearson product-moment coefficient, the Gaussian copula, the Gaussian copula-EVT, the Gumbel copula, the Gumbel copula-EVT, the Clayton copula, and the Clayton copula-EVT. The results suggest that the Clayton copula-EVT has the best performance regardless of the shapes of the return distributions.

JEL classifications: G15, F31, C46

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1 Introduction

Due to a lower correlation with industrial markets and higher return opportunities, investment in emerging markets has gained much attention recently. In the past few decades, a wave of financial liberalization in emerging markets has attracted enormous portfolio capital flows. According to Singh and Weisse (1998), the annual average private capital flow to developing countries increased to \$107.6 billion, during the period of 1989 through 1995, whereas during the period 1983 to 1988, it only increased by \$15.1 billion. After 2000, several Asian financial markets released foreign investors' maximum holding volume restrictions, making them even more attractive.

For U.S. investors, their overseas portfolio investments involve two transactions. The first transaction involves converting U.S. currency into the foreign local currency to invest in the foreign stock market. The second transaction occurs when the investors are ready to withdraw the foreign investments and sell the foreign shares, converting the value back to U.S. currency. In other words, their foreign portfolio investment returns comprise returns from foreign equity markets and the returns from exchange fluctuations. If the correlation between returns from foreign stock markets and the currency exchange fluctuations is positive, investors may face higher market risks, but also higher returns. Thus, when foreign stock markets are blooming, U.S. investors' foreign equity investments will bring greater rewards because the rewards are not just from equity markets but also from foreign currency appreciation. In the same manner, investors will suffer more if foreign stock markets are slumping, because, at the same time, foreign currency depreciates as well. In contrast, a negative correlation may dilute U.S. investors' losses or revenues, but investors' market risk will be lower, because a negative correlation means there may be a positive return from the equity markets and a negative return from the exchange markets at the same time, or vice versa. Consequently, an appropriate approach to estimate the correlation between stock market returns and exchange rate fluctuations is important for investors to measure their market risk.

Value-at-Risk (VaR) is currently one of the most frequently used tools used to estimate market risk. VaR provides information on how the loss of a portfolio can worsen over a target horizon with a given level of confidence. This approach presents the entire potential loss of a portfolio using a simple number that investors can easily adopt as a measure of market risk. Due to its convenience, financial regulators also use VaRs as a benchmark to determine financial institutions' regulatory capital requirements against market risk¹.

A popular VaR calculation approach is the Monte Carlo method, which suggests computing VaR based on the returns from the Monte Carlo simulation. The essential point for successfully implementing the Monte Carlo approach in computing the VaR of a portfolio depends on whether the investors can appropriately estimate the joint distribution of asset returns in the portfolio. Previously, correlations of financial asset returns were estimated based on the Pearson product-moment coefficient approach, which suggests that the returns of financial assets are normally distributed and the relationships between the financial assets are linear. However, recent studies imply that the foreign exchange rate and the stock indices have non-linear, time-varying correlations. The correlations are asymmetric across downside and upside markets movements, and the tails of the financial return distributions are fatter than that of a normal distribution. Therefore, the measurements of correlation, based on the normal distribution, are inappropriate and cause misleading results (Boyer et al., 1999; Longin and Solnik, 2001; Ang and Chen, 2002; Tastan, 2006; Tai, 2007; and Kolari et al., 2008).

In the past few years, researchers have suggested that copula functions can be powerful tools for modeling the correlation between the data in the characterization of nonlinearity and the asymptotic dependence (Longin and Solnik, 2001; Chen and Fan, 2006; Patton, 2006 a and b; Kole et al., 2007; and Rodriguez, 2007). Copulas involve joining the multivariate distribution functions of standard uniform random variables, and can be used to construct one-dimensional marginal distribution functions in a flexible way. Copulas have been broadly

¹According to the Basel Committee on Banking Supervision's regulatory (Basel I and II Accords), VaR is the preferred approach to measure market risks.

applied in the finance field in areas such as modeling joint loss distribution for risk management and modeling the correlation structure for multiple options pricing. Moreover, Since the tails of return distributions are fatter than that of a normal distribution, studies assert that the methodology derived from the extreme value theory (EVT) to model the tails of a multivariate distribution can provide a better measurement measurement in regards to tail behaviors (Da Silva and Mendes, 2003; Ho et Al., 2000; and Neftci, 2000).

In this paper, I propose to model the correlations between stock index returns and foreign exchange returns using the copula-EVT-based semiparametric approaches as demonstrated in Carmona (2004). More specifically, the copula-EVT semi-parametric approaches combine a parametric approach with the generalized Pareto distribution (GPD) in the tails and a nonparametric approach with the empirical distribution in the center of each asset return distribution, and apply copula functions to model the dependence between the two return distributions. To demonstrate the advantage of the copula-EVT approach, seven approaches are used to estimate the correlation coefficients: the Pearson product-moment coefficient, the Gaussian copula, the Gumbel copula, the Clayton copula, the Gaussian copula-EVT, the Gumbel copula-EVT, and the Clayton copula-EVT, and back testings are adopted to test the performance of the Monte Carlo VaRs from the seven correlation estimation approaches. Moreover, to verify whether the copula-EVT approaches can provide better performance in terms of measurement even with the data which do not exist significant fat tails in the distribution tails, such as some Asian emerging markets, six countries, Indonesia, Malaysia, Korea, Singapore, Taiwan, and Thailand, were selected based on their different tail shapes of the stock and exchange return distributions.

Since 2000, several Asian stock markets entered the post-financial liberalization period by opening their stock markets fully. Their market structural have considerably changed and it is worth reexamining their market situations. Most existing empirical studies of copula applications have focused on developed markets, while few have focused on Asian markets, and none have paid attention to the correlation between stock markets and currency exchange

markets. To the author's knowledge, this article is the first in the literature to focus on the correlation between stock returns and foreign exchange returns with daily data from six Asian countries in the post-financial liberalization period (2000 to 2007).

The rest of this paper is organized as follows. The next section explains portfolio returns and their VaRs, Section 3 reviews literature studies on the dependence between stock index returns and exchange rate fluctuations. Section 4 outlines the methodological approach, Section 5 reveals data statistics, Section 6 discusses the empirical results and Section 7 concludes.

2 Portfolio Returns and Their Value at Risk

In the absence of capital control with low transaction costs, the returns of overseas portfolio investment can be described as:

$$R_t = \frac{i_t \times e_t}{i_{t-1} \times e_{t-1}}$$

$$\text{Therefore, } LN(R_t) = LN[i_t - i_{t-1}] + LN[(e_t - e_{t-1})]$$

$$\text{To simplyfy, } r_t = r_{i,t} + r_{e,t}$$

Where R stands for the portfolio return and r is its log form. The subscripts t and t-1 are used to represent time t and time t-1. Therefore, r_t can be interpreted as the portfolio return at time t, which is defined as the log asset value differential between time t and t-1; i is the foreign stock index, and e represents currency exchange rate, which is defined as the price of one unit of foreign currency in U.S. dollars. Thus, a positive value of $LN(e_t - e_{t-1})$ denotes foreign currency appreciation against the U.S. currency. Since the initial investment value in the stock index is exactly the same as the initial value in the exchange market, r_t can

Table 1: Historical One Day VaR

Country	1% VaR	Expected loss
Indonesia:	-5.2148%	-50,811.17
Korea:	-5.6168%	-54,619.26
Malaysia:	-2.9486%	-29,055.44
Singapore:	-3.4052%	-33,478.75
Taiwan:	-4.5969%	-44,928.02
Thailand:	-4.2011%	-41,140.64

be treated as an equal weighted portfolio with the combination of $r_{i,t}$ and $r_{e,t}$. Therefore, understanding the correlation between $r_{i,t}$ and $r_{e,t}$ is very important when investors measure market risk of their portfolio investments.

One popular approach to evaluate market risk is Value-at-Risk (VaR). Given a positive value α close to 0, and over a target horizon h period (the differential period between time t and time $t+h$), VaR can be defined as the possible loss of the portfolio at the confidence level $(1-\alpha)$:

$$VaR_{\alpha}^h = \inf \{r \in \mathbb{R} \mid P(R \geq r) = 1 - \alpha\}$$

Where: R is a sequence number, $r_t, r_{t-1}, r_{t-2}, \dots, r_{t-h}$, stands for the portfolio rate of return at time $t, t-1, t-2, \dots, t-h$, respectively, and r is the VaR. The equation describes during h periods, the chance for the return being equal or less than r is α^2 . Empirically, 1% and 5% VaRs are commonly used. Table one illustrates historical 1% 1day VaR of Indonesia, Malaysia, Korea, Singapore, Taiwan, and Thailand. Among the six countries, Korea has the highest VaR at -5.6168%, and Malaysia has the lowest VaR at -2.9486%. The number means there is a 1% probability that in Korean stock market investors may lose 5.6168% in a day, while in Malaysian stock market, the loss is 2.9486%. In other words, if the investment amount is US\$ 1 million, the loss is \$54,619.26 in Korean and 29,055.44 in Malaysia³.

²This approach is defined as unconditional VaR, or historical VaR, since the VaR are computed based on historical return distribution.

³These numbers are calculated by continuously compounding. i.e. in Korean Stock Market: $\$1,000,000 \times (e^{-5.6168\%} - 1)$.

Although historical VaR is easy to apply, it has very limited applicability: the data range is constrained by the original data and may not be able to be extended to future periods. A more flexible approach to compute VaR is Monte Carlo VaR. By using the Monte Carlo method, VaR would be estimated with the return data simulated repeatedly from the random process, based on an appropriate joint distribution that accurately describes bilateral correlations among all asset returns. The Monte Carlo process is defined as:

$$\frac{dv}{v} = \mu dt + \sigma dx$$

Where: v stands for the asset value. The rate of return is $\frac{dv}{v}$, and V is defined as a sequence of $v_t, v_{t-1}, v_{t-2}, \dots, v_{t-n}$, the asset value on time $t, t-1, t-2, \dots, t-n$. μ is the expected asset return, σ is the volatility of the asset value, dt stands for the time differential and dx is a random process that can be defined as $\phi\sqrt{(dt)}$, where ϕ is drawing from its return distribution. In a two assets portfolio, the Monte Carlo process can be rewritten in matrix form with the subscript 1 and 2 to identify assets 1 and 2.

$$\frac{dv}{v} = \begin{bmatrix} \frac{dv_1}{v_1} \\ \frac{dv_2}{v_2} \end{bmatrix}, \mu dt = \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} dt, \sigma dx = \Phi \begin{bmatrix} \sqrt{(dt)} \\ \sqrt{(dt)} \end{bmatrix}$$

in which $\Phi = \begin{bmatrix} \sigma_1^2 & \rho_{1,2} \\ \rho_{1,2} & \sigma_2^2 \end{bmatrix}$. $\rho_{1,2}$ is the correlation coefficient of V_1 and V_2 . Therefore, the accuracy of the simulation strongly depends on the accuracy of the correlation coefficient of V_1 and V_2 ($\rho_{1,2}$).

3 Dependence between Stock Index Returns and Exchange Rate Fluctuations

Theoretical foundations explain the dependence between exchange rates and equity returns from a microeconomic and macroeconomic perspectives. From the microeconomic point of

view, home currency appreciation causes exporting firms to have fewer competitive advantages, lowering their revenues and stock prices; thus, the dependence between stock returns and the foreign exchange rate fluctuations is negative⁴. On the other hand, importing firms benefit from home currency appreciation through lower costs of materials, greater revenues, and higher stock prices, which suggests a positive dependence. From the macroeconomic point of view, when the domestic interest rate rises, the demand for home currency increases, leading to appreciation. However, domestic firms suffer from increased costs of capital due to higher interest rates and decreased stock values. Therefore, the dependence between stock returns and exchange rate changes should be negative.

Empirically, attempts to investigate the correlation between the stock market return and the exchange rate movements have been documented with mixed results being reported. Solnik (1987) applied the ordinary least squares (OLS) estimation to monthly and quarterly data for the period of July 1973 to December 1983 for eight industrial markets. His empirical results suggest that for monthly data a weak positive relationship exists between real stock return differentials and changes in the real exchange rate. This was found to be especially true during the period of 1979 to 1983. However, for quarterly data, a negative relationship was found. Ajayi and Mougoue's (1996) employed error correction model (ECM) to examine short term and long term dynamic correlations between stock prices and the exchange rate for "The Big Eight stock markets"⁵. They concluded that, in the short-run, the relationship between domestic stock prices and domestic currency was negative. However, in the long-run, stock prices increased and had a positive effect on the domestic currency value. Parto et al. (2002) studied correlations between stock returns and the exchange rate fluctuations in 16 OECD countries for the period of 1980 to 1997. They reported that the countries with higher exports were more negatively correlated with exchange rate risks, especially when the currency depreciated. At the same time, the countries with more importers were

⁴Some studies define the exchange rate as the price of foreign currency against one U.S. dollar. In this case, the correlation between stock returns and the exchange rate changes were positive.

⁵The Big Eight industrial economies in 1990s were Canada, France, Germany, Italy, Japan, the Netherlands, the United Kingdom, and the U.S.

associated with a positive correlation. Moreover, several studies examined the relationship between stock returns and exchange rates at the firms' level. Doidge et al. (2006) proposed that firms with a high exposure to international sales have higher returns than firms with no international sales. In contrast, the study performed by Kolari et al. (2008) implied that firms that were highly sensitive to the foreign exchange rate risk tended to have lower returns.

After 2000, Asian emerging markets entered a post-financial liberalization period that relaxed foreign investment restrictions. Empirical data reveal that the stock market structure significantly changed, due to the large volume of foreign capital inflows. The results of earlier studies may not be appropriate to describe the current situations. Furthermore, the financial crisis of 1997⁶ that approved foreign exchange rates and national stock indices were non-linear and time-varying with fatter tails representing financial return distributions. Studies based on purchase power parity and normal distributions on financial returns may have caused misleading results (Tai, 2007; Tasthan, 2006). To cope with this difficulty, recent studies (e.g. Chen and Fan, 2006; Patton, 2006 a, and b; Kole, et al., 2007; Bartram, et al., 2007; and Rodriguez, 2007) suggested that the copula functions were powerful tools for modeling dependence.

Copulas, derived from the Latin word *copulare*, to connect or to join, were first introduced as early as 1959 by Sklar and have recently been rediscovered. Copulas join multivariate distribution functions of standard uniform random variables and can be used to construct one-dimensional marginal distribution functions. By applying copula applications, the dependence of nonlinearity variables can be examined with higher degrees of flexibility. Since the late 1990s, copulas have been broadly applied in finance field such as modeling joint loss distribution for risk management and modeling the correlation structure for multiple options pricing.

⁶In October 1997, the Hong Kong Hang Seng Index dropped 1483 points, sparking the crash in the U.S. stock market, with a 554.26 point decrease in the Dow Jones Industrial Average. In December 1997, the South Korean Won depreciated over 60% since July 1, setting off the financial turmoil in the national stock market experiencing a 50.3% freefall (Granger et al.(2000)).

4 Methodological Approach

In this section seven correlation coefficient estimation methods are introduced.

4.1 Pearson's product-moment coefficient

This is a traditional approach used to estimate the correlation coefficient between two variables. Based on the assumptions such as that V_1 and V_2 are linear correlated, and normal distributed, this approach was defined as:

$$\rho_{1,2} = \frac{E[(V_1 - \mu_1)(V_2 - \mu_2)]}{\sigma_1\sigma_2}$$

4.2 Copulas

Copulas join multivariate distribution functions of standard uniform random variables and can be used to construct one-dimensional marginal distribution functions. Cherubini et al. (2004) stated that "the copula methodology has become the most significant new technique to handle the co-movement between markets and risks in a flexible way". Copulas were developed based on Sklar's dependency theory, which included several underlying functions: the marginal cumulative distribution functions (CDF) and a joint CDF. If we let U_1, U_2, \dots, U_n , be n random variables with a joint distribution function, then the joint distribution function can be presented as:

$$F_{U_1, U_2, \dots, U_n}(u_1, u_2, \dots, u_n) = P(U_1 \leq u_1, U_2 \leq u_2, \dots, U_n \leq u_n) \text{ for } (u_1, u_2, \dots, u_n) \in \mathbb{R}^n.$$

The marginal distribution functions of U_1, U_2, \dots, U_n , are

$$F_{U_1}(u_1) = P(U_1 \leq u_1)$$

$$F_{U_2}(u_2) = P(U_2 \leq u_2)$$

and

$$F_{U_n}(u_n) = P(U_n \leq u_n)$$

respectively. According to Sklar's theorem, if $F_{U_1}, F_{U_2} \dots F_{U_n}$, are continuous functions, then there exists a unique copula such that

$$F_{U_1, U_2 \dots U_n}(u_1, u_2, \dots, u_n) = C(F_{U_1}(u_1), F_{U_2}(u_2), \dots, F_{U_n}(u_n)). \quad (1)$$

Since most copulas are concerned with bivariate data, the above equation can be rewritten as

$$c(x, y) = P(X \leq x, Y \leq y), \text{ for } (x, y) \in [0, 1]^2$$

where $X = F_{U_1}(u_1)$ and $Y = F_{U_2}(u_2)$ ⁷. Studies suggest several parametric families of copulas, some of which are featured in tail dependence. On the basis of their different features, three types of copulas were used to examine the correlation coefficient: Gaussian, Gumbel, and Clayton. The Gaussian copula represents data that is normally distributed; The Gumbel copula shows a heavy tail on the right side; and The Clayton copula exhibits a fat tail on the left.

4.2.1 Gaussian Copula

The Gaussian copula represents normally distributed data. It is described as follows:

$$\begin{aligned} C(x, y) &= \int_{-\infty}^{\Phi^{-1}(x)} dx \int_{-\infty}^{\Phi^{-1}(y)} dy \frac{1}{2\pi\sqrt{1-\delta^2}} \exp\left\{-\frac{x^2 - 2\delta xy + y^2}{2(1-\delta^2)}\right\} \\ &= \Phi_{\delta}(\Phi^{-1}(x), \Phi^{-1}(y)) \end{aligned} \quad (2)$$

⁷Detailed derivation can be found in Cherubini, Luciano, and Vecchiato(2004), and Nelson (2006).

Where Φ denotes the distribution function of the univariate standard normal distribution and Φ_δ is the distribution function of the bivariate standard normal distribution with the correlation coefficient $-1 \leq \delta \leq 1$. Although the Gaussian copula can generate the bivariate standard normal distribution only if the margins are standard normal, the model is widely used in financial applications such as J.P.Morgan's RiskMetricsTM system (Zivot and Wang (2006)).

4.2.2 Gumbel Copula

The Gumbel copula is the most common extreme value copula. It describes upper tail dependence.

$$C(x, y) = \exp \left\{ - \left[(-\ln(x))^\delta + (-\ln(y))^\delta \right]^{\frac{1}{\delta}} \right\}, \delta \geq 1 \quad (3)$$

Where δ is the parameter used to control the strength of dependence. That is, if $\delta = 1$, then no dependence exists; if $\delta = +\infty$, then there exists perfect dependence. The coefficient of the dependence of the upper tail equals:

$$\lambda_u = 2 - 2^{1/\delta}$$

4.2.3 Clayton Copula

The Clayton copula describes lower tail dependence. It has the following form:

$$C(x, y) = \max \left[(x^{-\delta} + y^{-\delta} - 1)^{-\frac{1}{\delta}} \right] \quad (4)$$

Where $0 \leq \delta \leq \infty$. The coefficient of the lower tail dependence is defined as:

$$\lambda_l = 2^{-1/\delta}$$

4.3 Extreme Value Theory

From 2000 to 2007, the global financial markets experienced several marvelous incidents, such as the September 11 tragedy in the U.S. in 2001, the severe acute respiratory syndrome (SARS) in the South East Asia in 2003, and a tsunami in Indonesia, Sri Lanka, India, and Thailand in 2005. Each of these incidents caused huge market movements in the Asian financial markets; thus, when modeling dependence, the behavior of the distribution in the tails merits more attention. Modeling dependence by empirical distributions which traditional used for copula functions may not be sufficient as the proxies of tail behaviors. The literature suggests methodology derived from the extreme value theory (EVT) to model the tails with generalized Pareto distribution (GPD) can provide better measurements. Longin (2000) suggested a new approach that considered extreme values to calculate the VaR of a market position. He asserts that the advantages of the extreme value method include computations for high probability values, reduction of model risks, and consideration of event risks. Neftci (2000) also stressed that the extreme distribution theory performs well in capturing both the rate of occurrence and the extent of the effect of extreme events in financial markets. Da Silva and Mendes (2003) adopted EVT to examine the performance of Asian stock markets, identifying which type of extreme value asymptotic distribution better fits the historically extreme market events. Ho et al. (2000) applied this approach to value-at-risk measures (VaR) by modeling the tails of the return distributions of six Asian financial markets during volatile market conditions. They concluded that the VaR generated within this structure differed from those generated by variance-covariance and historical methods.

According to extreme value theory⁸, extreme values can be statistically modeled by two different methods: the block maxima method and the peaks-over-threshold method. The first approach separates the entire data period into several sub-periods and defines extreme events as the maximum or minimum value in each sub-period. The second method models the largest (smallest) values over a high (low) threshold. The first approach wastes a great

⁸See Coles (2001) and Beirlant et al. (2004) for detailed treatments of the extreme value theory.

deal of data and ignores the clustering phenomenon, which is commonly found in financial data. Thus, this study adopts the second method, which focuses on incidents exceeding a specified threshold.

4.3.1 Generalized Pareto Distribution

Consider X , a sequence of iid random variables $x_1, x_2, x_3, \dots, x_n$, representing the profits and losses of daily returns. The excess distribution y , which X exceeds a fixed threshold u , has the following cumulative distribution function:

$$F_u(y) = P(X - u \leq y \mid X > u) = \frac{F(y + u) - F(u)}{1 - F(u)}, \text{ for } 0 \leq y \leq x_0 - u \quad (5)$$

Where x_0 is the right endpoint of F . According to Picklands-Balkema-De Haan theorem, an appropriate distribution to approximate $F_u(y)$ is the generalized Pareto distribution (GPD). The Picklands-Balkema-De Haan theorem shows that:

$$\lim_{u \rightarrow x_0} \sup_{0 \leq y \leq x_0 - u} |F_u(y) - G_{\xi, \beta(u)}(y)| = 0$$

In other words, when u is very close to the endpoint, the excess distribution is the approximation of the GPD.

$$\widehat{F_u(y)} \approx \widehat{G_{\xi, \beta(u)}(y)} \quad (6)$$

The GPD has the following analytical form:

$$G_{\xi, \beta}(x) = \begin{cases} 1 - (1 + \xi \frac{x-u}{\beta})^{-\frac{1}{\xi}} & \text{for } \xi \neq 0 \\ 1 - \exp(-\frac{x-u}{\beta}) & \text{for } \xi = 0 \end{cases}, \beta \geq 0 \quad (7)$$

$$x \in D(\xi, \beta) = \begin{cases} [0, \infty), \xi \geq 0 \\ [0, \frac{-\beta}{\xi}], \xi < 0 \end{cases}$$

Where β is the scale parameter and ξ is the shape parameter. If $\xi < 0$, the tail is finite,

and $G_{\xi,\beta}(x)$ is the Weibull type distribution such as the beta and uniform distributions. If $\xi = 0$, then the tail declines exponentially (thin-tailed), and $G_{\xi,\beta}(x)$ is in the Gumbell family such as the normal, log normal, and exponential distributions. If $\xi > 0$, then the tail declines slowly (fat-tailed), and $G_{\xi,\beta}(x)$ is in the Fréchet family such as the Pareto distribution. The consensus of heavy tails makes the Fréchet family the relevant case.

4.3.2 Tail Estimation

Setting $x=y+u$ and using Equations (2) and (3), an approximation to the tails of the extreme distribution $F(x)$ for $x>u$ is defined as follows:

$$F(x) = (1 - F(u))G_{\xi,\beta}(x) + F(u) \quad (8)$$

The first term of Equation (4) may be estimated non-parametrically using the random proportion of the data in the tail.

$$\widehat{F(u)} = \frac{(n - k)}{n} \quad (9)$$

Where k denotes the number of observations beyond the threshold u . The choice of threshold value u is crucial, since it is an unpleasant trade-off between variance and bias. To fulfill the requirement of the Picklands-Balkema-De Haan theorem, one should choose as high as possible value of u . However, if u is too high, few exceedance data is available, leading to the high variance of estimators. In contrast, if u is too low, then the examination process will include observations not belonging to the tails, leading to biased estimators, and the Picklands-Balkema-De Haan theorem will not hold.

There is no common agreement on which method is best for selecting the threshold. While Gavin (2000) suggests an arbitrary threshold level of 90%, Neftci (2000) defines the threshold as 1.645 of the unconditional variance of the data, which represents 5% of the observations if the true data is normally distributed. Longin and Solnik (2001) use the

Monte Carlo simulation method to select the threshold by optimizing the trade-off between bias and variance. Frey and McNeil (2000) apply the mean-excess plot to choose the optimal threshold. This paper adopts Neftci's approach and slightly adjusts the value using the mean-excess plot approach to obtain the optimal thresholds. Combining Equations (3) and (5), $F(x)$ can be estimated with a GPD fitted by the maximum likelihood to obtain the tail estimator.

$$\widehat{F}(x) = 1 - \frac{k}{n} \left(1 + \widehat{\xi} \frac{x - \mu}{\widehat{\beta}} \right)^{\frac{-1}{\xi}}, \text{ for } x > u \quad (10)$$

Therefore, the other three correlation coefficient estimation approaches are the Gaussian copula-EVT, the Gumble copula-EVT, and the Clayton copula-EVT. They are semi-parametric approaches with an empirical distribution in the center and the GPD in the tails.

4.3.3 Parameter estimation

Parameters were estimated by two stages. In the first stage, the scale and sharp parameters were estimated, and the marginal distributions were modeled with the GPD in the tails and the empirical distribution in the center. Then, using the marginal distributions to model the joint distribution, I applied an inference function for the margins (IFM) method, as proposed by Joe (1997) to estimate parameters.

Recall that the bivariate joint density function can be represented as:

$$f(U_1, U_2; \eta) = C(F_{U_1}(u_1; \theta_1), F_{U_2}(u_2; \theta_2); \theta) f_{U_1}(u_1; \theta_1) f_{U_2}(u_2; \theta_2).$$

Where θ_1 and θ_2 are parameters for the marginal distribution F_{U_1} and F_{U_2} respectively, θ is the parameter for the copula density and $\eta = (\theta'_1, \theta'_2, \theta')$ are the parameters of the joint density. The exact log-likelihood function is then

$$l(\eta; U_1, U_2) = \sum_{i=1}^n \ln c(F_{U_1}(u_i; \alpha_1), F_{U_2}(u_i; \alpha_2); \theta) + \sum_{i=1}^n \ln f_{U_1}(u_i; \alpha_1) + \sum_{i=1}^n \ln f_{U_2}(u_i; \alpha_2)$$

According to Joe (1997), these set of parameters can be estimated in two steps:

First, the parameters of the marginal distributions were estimated as:

$$\hat{\theta}_1 \equiv \arg \max \sum_{i=1}^n \ln f_{U_1}(u_1; \theta_1)$$

$$\hat{\theta}_2 \equiv \arg \max \sum_{i=1}^n \ln f_{U_2}(u_2; \theta_2)$$

Then at the second step, given $\hat{\theta}_1$ and $\hat{\theta}_2$, the dependence parameter were estimated as:

$$\hat{\theta}_c \equiv \arg \max \sum_{i=1}^n \ln c(F_{U_1}(u_1; \alpha_1), F_{U_2}(u_2; \alpha_2); \theta)$$

5 Data Statistics

The data used in this study included daily closing stock indices and daily closing prices of US foreign currency exchange rates in six countries, Indonesia, Korea, Malaysia, Singapore, Taiwan, and Thailand, as reported by Global Financial Data. The stock indices applied were the Jakarta SE Composite Index of Indonesia, the Korea Stock Exchange Stock Price Index (KOSPI), the Kuala Lumpur Stock Exchange Composite of Malaysia, the Singapore Strait Times Stock Index, the Taiwan Stock Exchange Capitalization Weighted Index, and the Stock Exchange of Thailand General Index. The sample period was from the first business day in 2000 to the last business day in 2007⁹.

Table 2 summarizes the data statistics, which include the stock index returns, the currency exchange returns, and the portfolio returns. The first two columns present the mean and standard deviation. The data demonstrated high returns and high standard deviations, which is common in financial markets. The negative skew in the stock index returns for

⁹Since the six country has its own business days, the observations varied in each country. Indonesia had 1957 observations, Korea had 1968 observations, Malaysia had 1967 observations, Singapore had 2004 observations, Taiwan had 1997 observations, and Thailand had 1961 observations.

all six countries means that the distributions have long tails to the left. The Jarque-Bera (JB) test, which follows a chi-square distribution with two degrees of freedom, was used to determine normality. The null hypotheses of the stock index return, the exchange return, and the portfolio return series in all six countries were rejected, indicating that the return distributions were far from normal. The higher kurtosis also lead to a strong rejection in the JB test and confirmed that the distributions have non-normality.

Figure 1 to 6 illustrate daily stock indices and foreign exchange rates, daily stock index returns and foreign exchange returns, QQ plot¹⁰ of portfolio daily returns and daily portfolio return of the six countries respectively. QQ plot is a scatter plot constructed by the empirical quantiles (vertical axis) against the normal quantiles (horizontal axis), which can be used to test the normality of a distribution. The results of the QQ plots in the six countries also verified that the portfolio returns of the six countries are non-normally distributed.

Table 3 presents the expected annual returns of the foreign exchange, the stock index, and the portfolio in the six countries¹¹. Basically, the expected annual return of foreign exchange is lower than the expected return of stock index in the six countries. That is because most Asian countries impose currency control, and the governments are likely to keep the currency exchange rates stabilized; i.e., the Malaysia ringgit against the U.S. dollar kept around 3.799 from January 2000 to mid 2005. Although Indonesia has the highest expected return of stock index, its expected return of portfolio is only 14.9785%, because the expected return of foreign exchange of -3.439% offsets the return from the stock index. The expected portfolio returns in Korea, Malaysia, Singapore, and Thailand are higher than their expected stock index returns because the positive returns in foreign exchange make the portfolio returns even greater. In Taiwan, U.S. investors will suffer more since both negative returns in stock index and in foreign exchange made the portfolio return even worse.

¹⁰Q stands for quantile.

¹¹The expected annual return is computed by assuming that there are 250 business days in a year. Therefore, the annual expected return is $e^{mean \times 250} - 1$.

Table 2: Data Summary and Statistics

Country		Mean	Std. Dev.	Skewness	Kurtosis	Jarque-Bera
Indonesia:	Foreign Exchange:	-0.00014	0.007771	1.039	24.79	***39068.85
	Stock Index:	0.0006982	0.01378	-0.7058	7.615	***1899.39
	Portfolio	0.0005583	0.01772	-0.6056	9.035	***3089.41
Korea:	Foreign Exchange:	0.00009226	0.004613	-0.4036	5.475	***555.59
	Stock Index:	0.0002962	0.01816	-0.5519	6.966	**1389.63
	Portfolio	0.0003885	0.01976	-0.5209	6.295	***979.04
Malaysia:	Foreign Exchange:	0.00007094	0.001428	2.093	19.79	***23116.07
	Stock Index:	0.00027953	0.009237	-0.5709	8.961	***3019.51
	Portfolio	0.0003504	0.009598	-53.09	8.409	***2490.30
Singapore:	Foreign Exchange:	0.00006787	0.00267	0.0554	5.927	***716.60
	Stock Index::	0.0001438	0.01155	-0.5157	7.668	***1908.35
	Portfolio	0.0002117	0.01208	-0.4713	6.784	***1269.55
Taiwan:	Foreign Exchange:	-0.00002261	0.003397	0.4196	95.08	***705855.05
	Stock Index:	-0.00002099	0.01553	-0.1011	4.945	***318.37
	Portfolio	-0.00004359	0.01638	-0.1276	4.733	***255.514
Thailand:	Foreign Exchange:	0.0001121	0.00599	0.8475	137.7	***1481785.00
	Stock Index::	0.000277	0.0145	-0.7406	13.53	***9340.70
	Portfolio	0.0003891	0.01606	-0.435	10.67	***4864.47

*0.90 statistical significance, **0.95 statistical significance, ***0.99 statistical significance

Table 3: Expected Annual return

Country	Expected Annual Return	
Indonesia:	Foreign Exchange:	-3.439%
	Stock Index:	19.071%
	Portfolio	14.9785%
Korea:	Foreign Exchange:	2.3333%
	Stock Index:	7.6861%
	Portfolio	10.1998%
Malaysia:	Foreign Exchange:	1.7893%
	Stock Index:	7.2382%
	Portfolio	9.1551%
Singapore:	Foreign Exchange:	1.7112%
	Stock Index:	3.6604%
	Portfolio	5.4351%
Taiwan:	Foreign Exchange:	-0.564%
	Stock Index:	-0.523%
	Portfolio	-1.084%
Thailand:	Foreign Exchange:	2.8421%
	Stock Index:	7.1704%
	Portfolio	10.2163%

6 Empirical results

The estimations of the return distribution tails are presented in Table 4. In Taiwan and Korea, the sharp parameters of the stock index return distributions' upper tails are significantly negative, indicating the upper tails are finite. A possible explanation is that both Korean and Taiwanese stock markets impose daily price movement limitation policy¹². When a bull market occurs, stocks are not allowed to be traded at the prices over the limit, making the stock return distribution's upper tail truncated. As for the lower tail distribution of stock index returns, Taiwan still has a truncated lower tail at 90 percent statistical significance, while in Korea, there is no significant evidence to prove the shape of the lower tail of the stock index return distribution. Thailand has significant fat tails on both upper and lower sides because the limit of daily stock price movements is 30 percent in Thailand, which gives the stock prices more space to shift. The shape of the upper tail of the stock index return distribution in Indonesia, Malaysia, and Singapore is not significant, but the lower tail of the stock index return distribution in these three countries is fat at statistically significant level.

Both upper and lower tails of the foreign exchange return distribution in Korea are truncated, implying government intervention occurs when the price of the Korean won jumps too high or drops too low. Taiwan and Thailand have significant fat tails on both upper and lower sides of the foreign exchange return distribution, while Indonesia, Malaysia, and Singapore only have significant upper fat tails, but there is not enough evidence to support that their lower tails are also fat.

Table 5 reveals the correlation coefficients estimated from the seven approaches: the Pearson correlation, the Gaussian copula, the Gaussian copula-EVT, the Gumbel copula, the Gumbel copula-EVT, the Clayton copula, and the Clayton copula-EVT. The results suggest that the stock index returns are weakly positively correlated with the foreign ex-

¹²Daily stock price movement is limited at 12 percent in the Korea Stock Exchange, and 7 percent in the Taiwan Stock Exchange.

Table 4: Estimation Results of Distribution Tails

Country	Indonesia	Korea	Malaysia	Singapore	Taiwan	Thailand
Stock Index Returns:						
ξ	0.0131	-0.2299	0.0468	-0.0027	-0.2788	0.2087
(t-Stat)	(0.1169)	**(-1.9414)	(0.4955)	(-0.0228)	***(-2.6621)	** (1.8101)
Upper tail β	0.0074	0.0147	0.0058	0.0075	0.0135	0.0063
(t-Stat)	*** (6.3511)	*** (5.8178)	*** (7.8426)	*** (6.1583)	*** (7.1298)	*** (6.7876)
llv	308.1	210.4	573.8	315.4	379.7	431.9
ξ	0.2505	0.2359	0.3383	0.1482	-0.1439	0.2787
(t-Stat)	** (1.8354)	(1.6891)	** (2.2113)	* (1.3776)	* (-1.3596)	*** (2.6082)
Lower tail β	0.0085	0.0104	0.0059	0.0078	0.0125	0.0085
(t-Stat)	*** (5.9346)	*** (5.7843)	*** (5.6042)	*** (6.8733)	** (6.7104)	*** (5.8577)
llv	338	303.1	398.3	366	313.5	292.72
Foreign Exchange Returns:						
ξ	0.2897	-0.1761	0.2465	0.2259	0.4383	0.5542
(t-Stat)	*** (2.5664)	** (-2.3007)	* (1.6092)	** (1.9080)	*** (3.8363)	*** (3.6700)
Upper tail β	0.0050	0.0034	0.00135	0.0012	0.0018	0.0027
(t-Stat)	** (7.1975)	*** (8.9451)	*** (5.5971)	*** (6.8782)	*** (7.1390)	*** (5.7995)
llv	480.8	696.5	598.7	525	602.9	435.3
ξ	0.0136	-0.2156	0.0149	0.1086	0.6394	0.4149
(t-Stat)	(0.1107)	** (-1.7427)	(0.1101)	(1.0154)	*** (3.7947)	*** (3.1819)
Lower tail β	0.0076	0.0047	0.0020	0.0015	0.0116	0.0033
(t-Stat)	** (6.2212)	*** (5.8190)	*** (5.7423)	*** (6.8173)	*** (5.3800)	*** (6.3579)
llv	360	306.1	432.7	509.1	463.3	460.7

*0.90 statistical significance, **0.95 statistical significance, ***0.99 statistical significance

change returns. In Indonesia, Korea, Malaysia, and Singapore, the correlation coefficients estimated by Gaussian copula are close to the correlation coefficients estimated by Pearson correlation approach. This is because the Gaussian copula approach is derived based on normal distribution assumption, which is the same assumption that Pearson correlation applies. However, in Taiwan and Thailand, the correlation coefficients estimated by the Gaussian copula are quite different from the correlation coefficients estimated by the Pearson correlation approach because, according to the results in the Table 4, Taiwan has significantly truncated upper and lower tails in its stock index return distribution and significantly fat upper and lower tails in its foreign exchange return distribution; therefore, the correlation between stock index return and foreign exchange return is not linear, and the correlation coefficient is quite different from that of the Pearson correlation. In Thailand, fat tails exist in upper and lower sides of both stock index return distribution and foreign exchange return distribution. In the same manner, the correlation between stock index return and foreign exchange return is not linear, and the correlation coefficient is quite different from that of the Pearson correlation.

For the tail dependence, the figures in the Gumbel copula or the Gumbel copula-EVT mean that when one asset has a higher positive rate of return on a day, the probability that the other asset will also yield a higher positive rate of return on the same day, i.e. In Indonesia, the correlation coefficient of the Gumbel copula is 20.72 percent, which means when the stock index yields a higher rate of return on a specific day, there is a 20.72 percent probability that the foreign exchange will also yield a higher rate of return on the same day. By the same fashion, the figures in the Clayton copula or the Clayton copula-EVT present the lower tail dependence. Therefore, if the stock market is slumped on a specific day in Indonesia, the probability for the Indonesia rupiah to plunge on the same day is 18.42 percent. By comparing the correlation coefficients among the Gaussian copula, the Gumbel copula, and the Clayton copula, the correlation coefficients of the Gumbel copula and the Clayton copula are smaller than that of the Gaussian copula, indicating that the tail dependence is

weaker. Moreover, in general, the correlation coefficients of the Gumbel copulas, the upper tail dependence, are higher than that of the Clayton copulas, the lower tail dependence, implying that the government stepped in especially when bear markets occurred. Among the six countries, the values of correlation coefficients in Indonesia are the largest. A possible explanation for this circumstance is that Indonesia does not impose foreign exchange control and the limit of the stock price movement is 30 percent, providing more freedom for both stock index and foreign exchange rate movements.

6.1 Backtesting

To examine the advantage of the copula-EVT methodologies in estimating the correlation coefficients, Backtestings were conducted to test the Monte Carlo VaRs generated by the correlation coefficients of the seven approaches. Assume at time t , \widehat{VaR}_α^h can be computed from a historical series, $r_t, r_{t-1}, r_{t-2}, \dots, r_{t-h}$. At time $t+1$, a violation is said to occur if $r_{t+1} < \widehat{VaR}_\alpha^h$ ¹³. According to McNeil and Frey (2000), If the indicator variances are defined as $I_{t+1} = I_{r_{t+1} < \widehat{VaR}_\alpha^h}$, the process $(I_t)_{t \in \mathbb{R}}$, is binomially distributed with violation probability α . In this paper, α is set at 1 percent, and h is 250, since there are roughly 250 business days in a year.

The backtesting results of the Monte Carlo VaRs are reported in table 6. Due to different return distribution shapes among the six countries, the backtesting results vary among the seven methodologies. By comparing the p values, in general, the copula-EVT approaches can provide better performance than others. The Pearson correlation approach reaches 99 percent significance in Singapore, and 95 percent significance in Malaysia. That is because these two countries do not include significant fat tails on their stock index return distribution or foreign exchange return distribution. When applying the Pearson product-moment coefficient approach in the country that has significant fat tails on its return distributions,

¹³In some VaR documents, this inequality is presented as $r_{t+h+1} > \widehat{VaR}_\alpha^h$ by assuming r_{t+h+1} is loss function and both r_{t+h+1} and \widehat{VaR}_α^h are in absolute value.

Table 5: Estimation Results of Copulas

Country	Indonesia	Korea	Malaysia	Singapore	Taiwan	Thailand
Person	0.2974	0.2361	0.1802	0.0852	0.1493	0.0666
δ	0.2857	0.2392	0.1659	0.0925	0.2334	0.1629
Gaussian (t- _{Sta})	***(14.2341)	***(11.5053)	***(7.6204)	***(4.1687)	***(11.2645)	***(7.4596)
llv	82.29	57.24	27.08	8.503	55.26	26.02
δ	0.2850	0.2738	0.1429	0.0913	0.2298	0.1600
Gaussian-EVT (t- _{Sta})	***(14.2684)	** (11.4877)	***(6.9313)	***(4.1354)	***(11.1347)	** (7.3665)
llv	82.89	57.22	22.91	8.374	54.27	25.45
δ	1.1873	1.1537	1.0814	1.0504	1.1430	1.1030
λu	0.2072	0.1764	0.1017	0.0655	0.1661	0.1254
Gumbel (t- _{Sta})	***(60.3273)	***(62.2950)	***(65.8997)	***(70.6829)	***(62.0077)	***(64.6103)
llv	66.48	47.98	16.51	7.094	38.74	24.32
δ	1.1852	1.1524	1.0781	1.0497	1.1403	1.1016
λu	0.2053	0.1752	0.0980	0.0646	0.1635	0.1239
Gumbel -EVT (t- _{Sta})	***(60.5464)	** (62.4653)	***(67.3352)	***(71.2186)	***(62.2600)	** (64.8894)
llv	65.5	47.49	15.98	7.033	37.78	24.02
δ	0.4098	0.3090	0.1818	0.1086	0.2685	0.1758
λl	0.1842	0.1061	0.0221	0.0017	0.0757	0.0194
Clayton (t- _{Sta})	***(11.9434)	*** (9.5011)	*** (6.2297)	*** (4.0050)	*** (8.4661)	*** (5.7942)
llv	97.96	59.33	24.13	9.631	45.56	20.58
δ	0.4030	0.3028	0.1274	0.1060	0.2589	0.1708
λl	0.1791	0.1013	0.0043	0.0014	0.0688	0.0173
Clayton-EVT (t- _{Sta})	***(11.8683)	** (9.4039)	*** (5.1229)	*** (3.9630)	*** (8.2757)	*** (5.6856)
llv	97.9	58.5	15.83	9.456	43.77	19.36

*0.90 statistical significance, **0.95 statistical significance, ***0.99 statistical significance

i.e., Thailand, this approach will not be able to provide an accurate estimate. Basically, the copula-EVT approaches have better performance in the six countries. In Indonesia, except for the Pearson correlation, the Gaussian copula, the Gaussian copula-EVT, the Gumbel copula, the Gumbel copula-EVT, the Clayton copula, and the Clayton copula-EVT approaches can provide accurate estimates of performance at a 99 percent significant level. The estimates provided by the Gumbel copula-EVT and the Clayton copula-EVT have better performance than other approaches in Korea, Malaysia, Singapore, and Taiwan. In Thailand, only the Clayton copula and the Clayton copula-EVT can provide more accurate estimates. To sum up, the Clayton copula-EVT approach has the best performance, which reaches 95 percent significance in Korea and 99 percent significance in Indonesia, Malaysia, Singapore, Taiwan, and Thailand.

Table 6: Back testing Results (P-Values)

Country	Indonesia	Korea	Malaysia	Singapore	Taiwan	Thailand
Pearson	*0.0730	*0.0764	**0.0437	***0.0012	*0.0542	0.2874
Gaussian Copula	***0.0005	0.7508	**0.0238	0.0954	0.3352	0.9767
Gaussian Copula-EVT	***0.0001	0.7508	**0.0123	0.0565	0.1487	0.4116
Gumbel Copula	***0.0005	0.5755	0.0759	**0.0170	0.1487	0.2874
Gumbel Copula-EVT	***0.0000	**0.0124	**0.0123	***0.0008	**0.0161	0.1212
Clayton Copula	***0.0001	*0.0764	0.1254	**0.0003	0.0376	*0.0730
Clayton Copula-EVT	***0.0000	**0.0124	**0.0028	***0.0000	***0.0001	***0.0057

*0.90 statistical significance, **0.95 statistical significance, ***0.99 statistical significance

7 Conclusion

This article examines the dependence between stock index returns and foreign exchange returns during the post-financial liberalization period for Indonesia, Korea, Malaysia, Singapore, Taiwan, and Thailand. Conventionally, correlation is estimated by the Pearson product-moment coefficient, which assumes that the relationship between two assets is linear. However, the existence of fat-tailed, skewed distributions of returns and a non-linear relationship between financial assets makes the conventional approach incongruous. To cope with this difficulty, this research applies copula-EVT based semiparametric approaches to model the distributions of stock index returns and foreign exchange returns, and the correlation between the two return distributions. In particular, the marginal distributions are modeled by a semiparameter approach in which the distribution center is modeled by a non-parameter empirical distribution, and the distribution tails are modeled by the generalized Pareto distribution (GPD) with parameters, and copula functions are used to build flexible models of the joint distribution of the two asset return distributions.

The Pearson product-moment coefficient, the Gaussian copula, the Gaussian copula-EVT, the Gumbel copula, the Gumbel copula-EVT, the Clayton copula, and the Clayton copula-EVT approaches are adopted to estimate the correlation coefficients between the stock index return distributions and the foreign exchange return distributions. To examine the advantage of the copula-EVT semi-parameter approaches when applying VaR computing, backtestings are applied to test the Monte Carlo VaRs generated with the correlation coefficients from the above seven methodologies. To verify if the copula-EVT approaches could perform well even with data that don't have strong fat tails of their distributions, Indonesia, Korea, Malaysia, Singapore, Taiwan, and Thailand are selected based on their different shapes of return distributions. In Thailand, both stock index return distribution and foreign exchange return distribution have upper and lower fat tails, while in Taiwan, the upper and lower tails of the stock index return distribution are truncated, and the upper and lower tails of the foreign exchange return distribution are fat. Indonesia, Malaysia, and

Singapore have fat lower tails of their stock index return distributions and fat upper tails of their foreign exchange return distributions. The upper tail of the stock index return distribution and both upper and lower tails of the foreign exchange return distribution in Korea are truncated.

The estimation results of the correlation coefficient suggest that although the six countries all have positive correlations between returns from stock index and currency exchange, the extent of these correlations is weak. Among the six countries, Singapore has the lowest correlation between its stock index return and foreign exchange return, while Indonesia has the highest correlation level. In comparison to the results of upper tail dependence, the dependence in the lower tail is relatively small. The results imply that developed markets have a long history of foreign capital inflows, which do not easily shift out; therefore, stock market movements may not cause exchange rate movements and vice versa. Moreover, in emerging markets, it is quite common to see government intervention, especially when the market is on a downside; thus, the results of lower tail dependence are smaller than those of upper tail dependence. Furthermore, except for Indonesia, the other five Asian countries impose exchange control, which may contribute to the weak correlations.

The backtesting results of the Monte Carlo VaRs reveal that the copula-EVT approaches can provide better performance. Although the Pearson product-moment coefficient approach has acceptable performance in a country that doesn't have significant fat tail or truncated tails on its return distributions, i.e., Singapore, in a country with significant fat tails on both its stock index return and foreign exchange return distributions, such as Thailand, the Pearson product-moment coefficient approach cannot provide an accurate estimate. In general, the copula-EVT approaches perform better than the copula approaches. Among the seven correlation estimation approaches, the Clayton copula-EVT approach has the best performance; it reaches the highest significant level in all six countries no matter the shapes of their return distributions.

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Figure 1: Indonesia: (a) Daily stock indices and foreign exchange rates; (b) Daily stock index returns and foreign exchange returns; (c) QQ plot of portfolio daily returns; (d) Daily portfolio return.

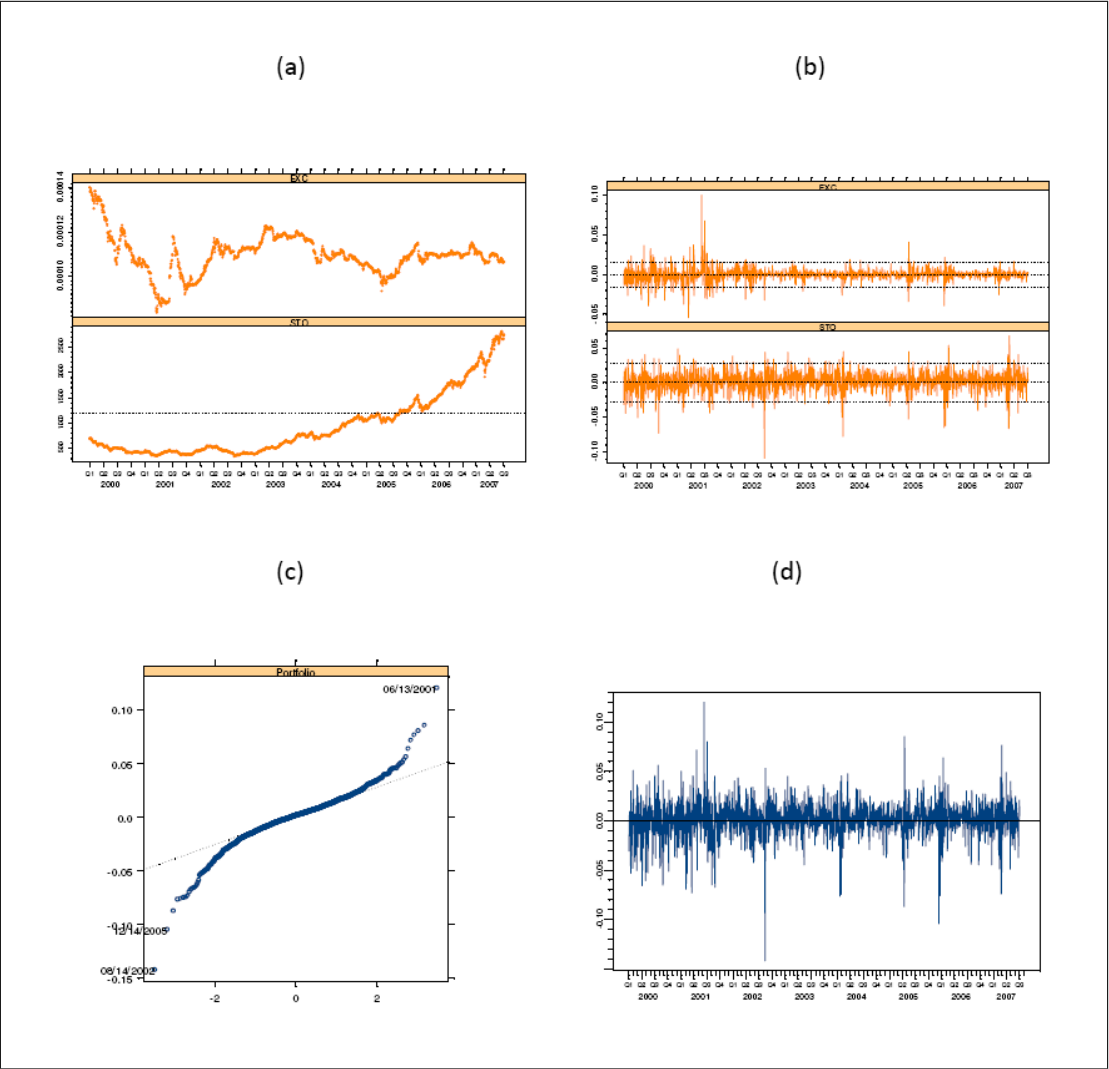


Figure 2: Korea: (a) Daily stock indices and foreign exchange rates; (b) Daily stock index returns and foreign exchange returns; (c) QQ plot of portfolio daily returns; (d) Daily portfolio return.

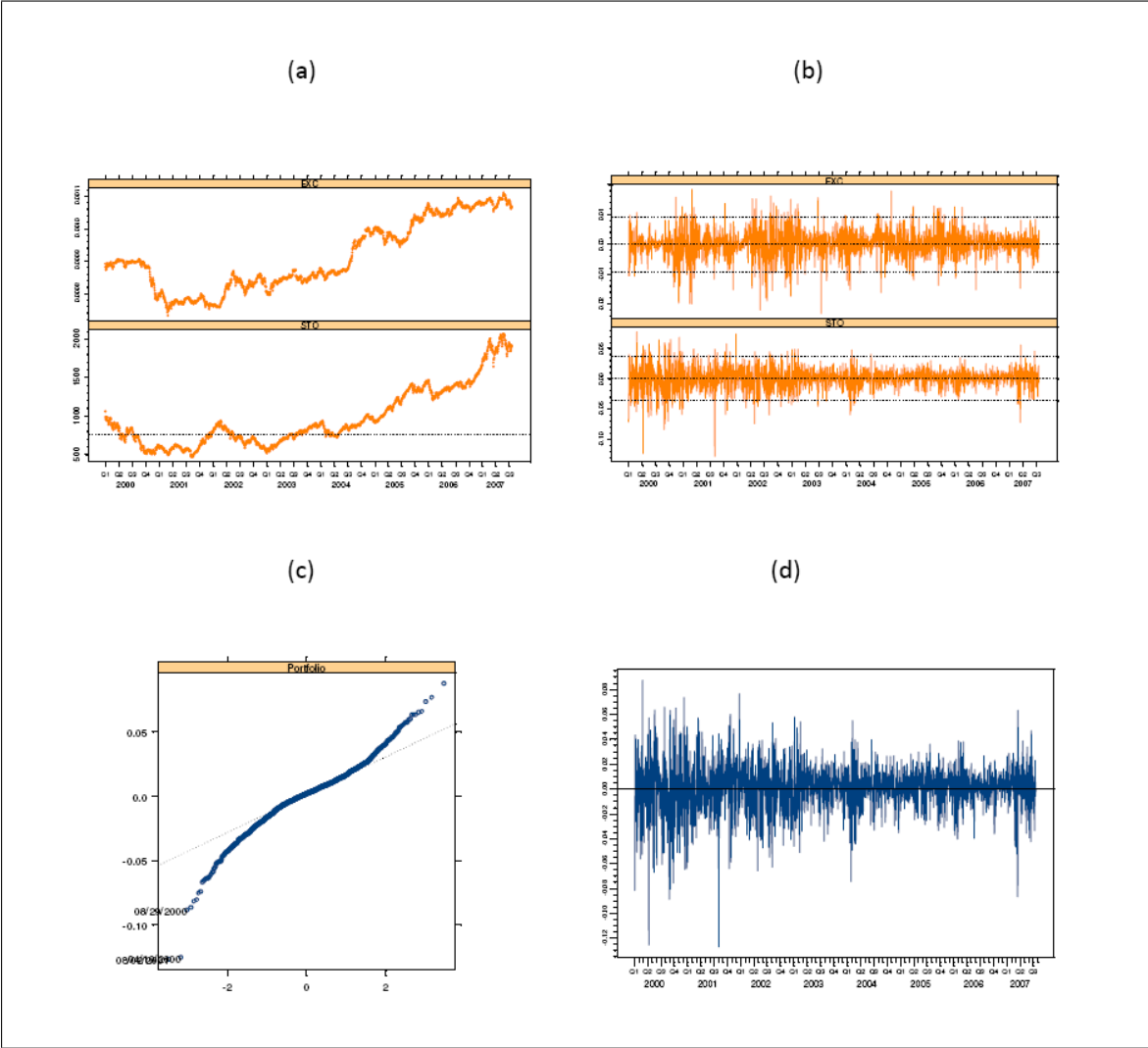


Figure 3: Malaysia: (a) Daily stock indices and foreign exchange rates; (b) Daily stock index returns and foreign exchange returns; (c) QQ plot of portfolio daily returns; (d) Daily portfolio return.

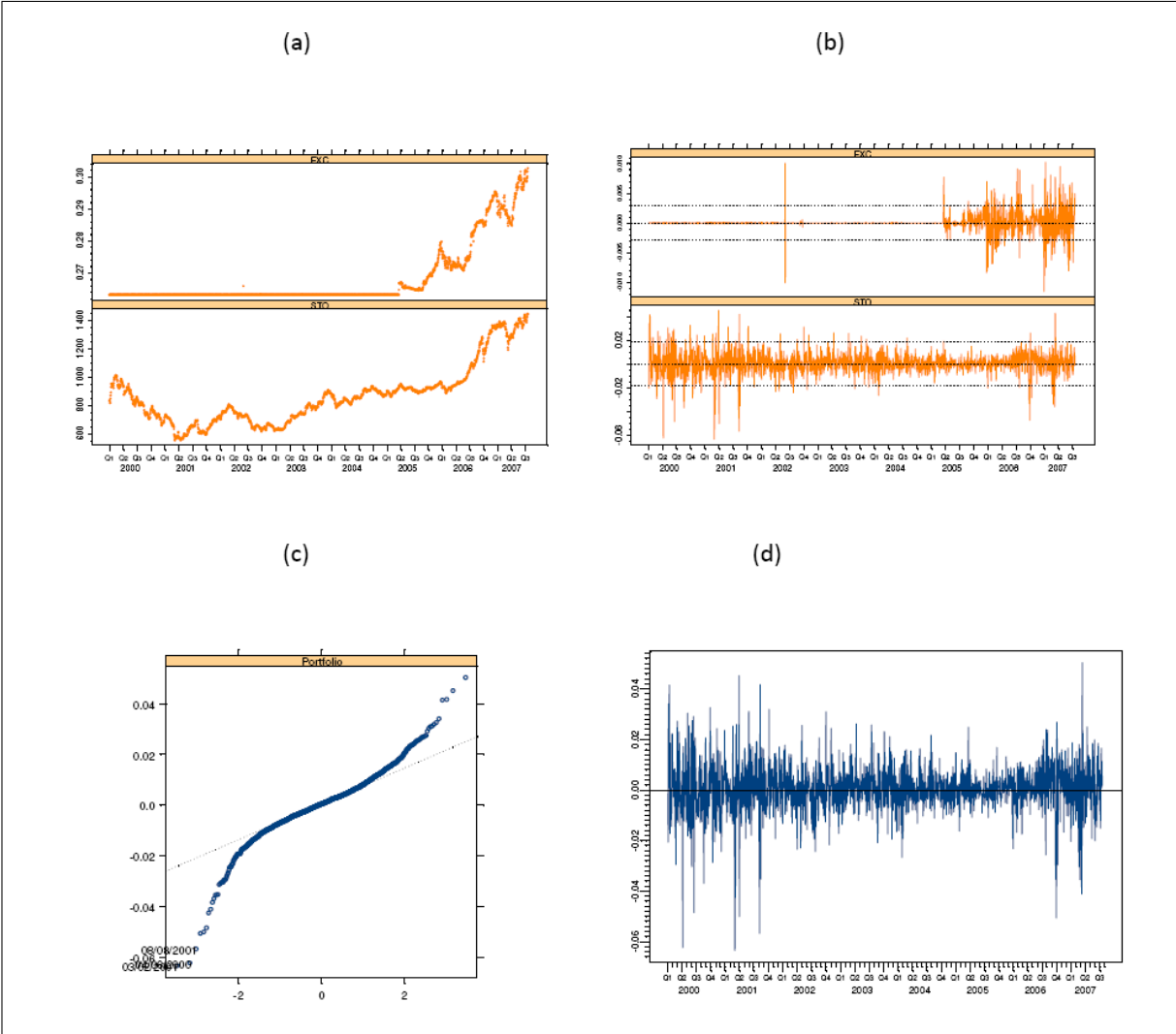


Figure 4: Singapore: (a) Daily stock indices and foreign exchange rates; (b) Daily stock index returns and foreign exchange returns; (c) QQ plot of portfolio daily returns; (d) Daily portfolio return.

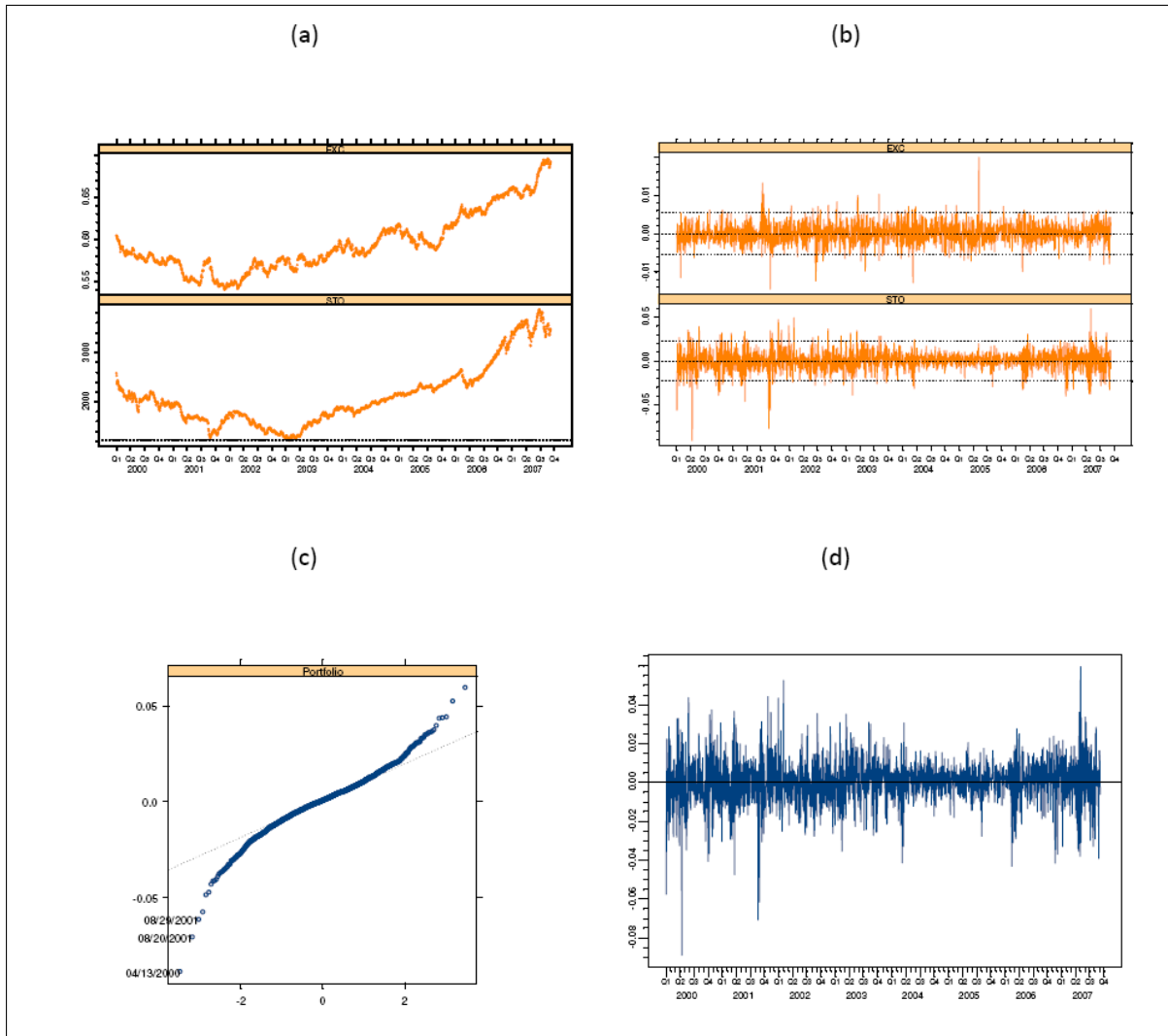


Figure 5: Taiwan: (a) Daily stock indices and foreign exchange rates; (b) Daily stock index returns and foreign exchange returns; (c) QQ plot of portfolio daily returns; (d) Daily portfolio return.

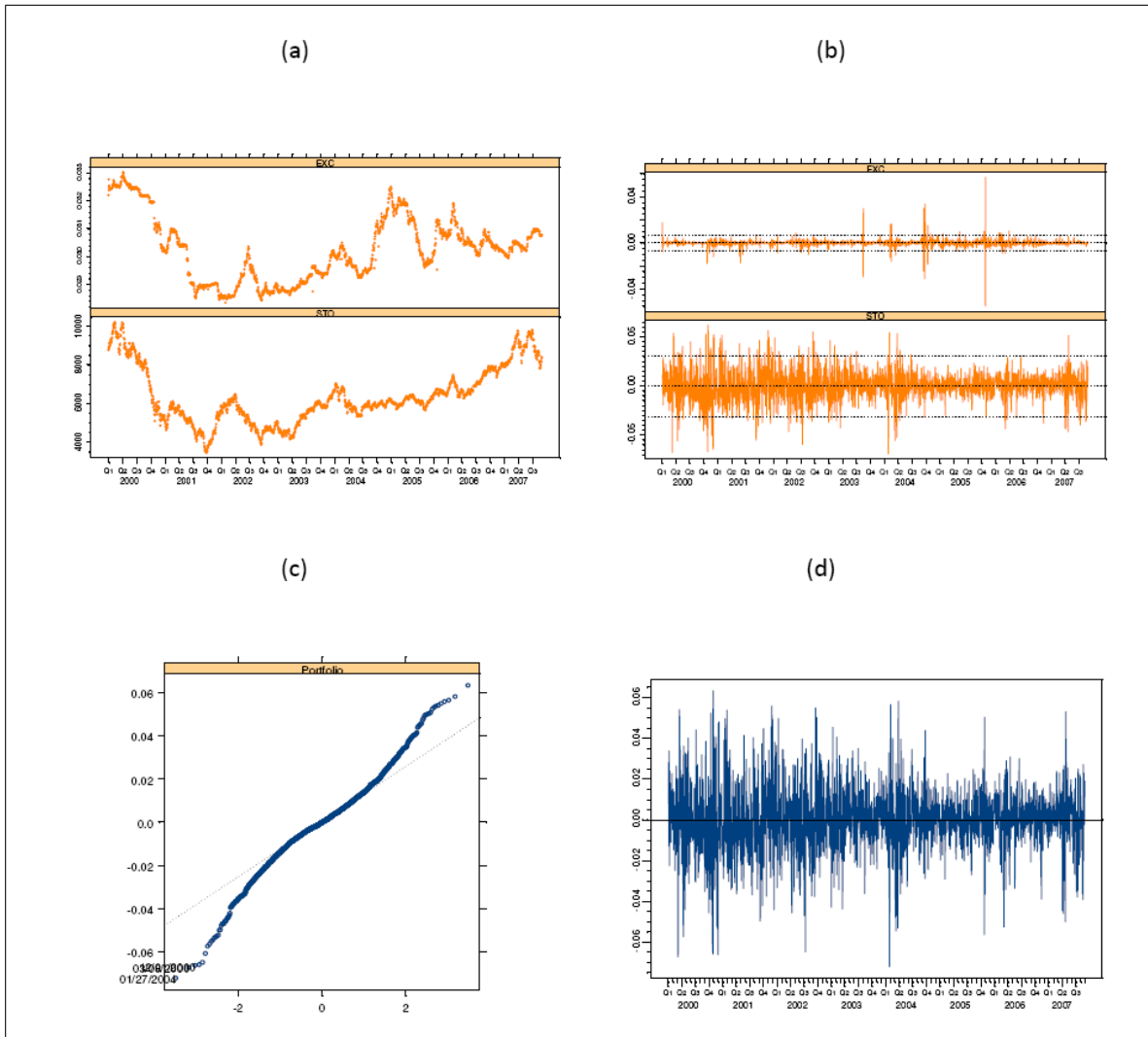


Figure 6: Thailand: (a) Daily stock indices and foreign exchange rates; (b) Daily stock index returns and foreign exchange returns; (c) QQ plot of portfolio daily returns; (d) Daily portfolio return.

