

Durable Goods Inventories and the Great Moderation

James A. Kahn¹

Federal Reserve Bank of New York

Preliminary and Incomplete

December 2007

¹New York University and the Federal Reserve Bank of New York. The views expressed in this paper are those of the author and do not necessarily reflect the views of the Federal Reserve Bank of New York or the Federal Reserve System. Please direct all correspondence to the author at: jkahn@stern.nyu.edu or Stern School of Business, Economics Department, New York University, 44 W. 4th Street, New York, NY 10012.

Abstract

The so-called .Great Moderation. was not an across-the-board uniform reduction in volatility, but was heavily concentrated in the durable goods sector. This paper presents evidence of the role of inventories in that sector's stabilization, and then provides a model that is consistent with the facts in that sector as well as with anecdotal accounts of improved inventory management. The key innovation in the model is to apply the idea of stockout-avoidance to a production-to-order industry, which best characterizes the durable goods sector. The key technological advance is characterized by a reduced lead time on materials orders. Numerical examples suggest that the mechanism of the model is capable of delivering reductions in the volatility of both output and sales that are on the order of magnitude of those observed in the data.

The so-called “Great Moderation” was not an across-the-board uniform reduction in volatility, but was heavily concentrated in the durable goods sector. This paper presents evidence of the role of inventories in that sector’s stabilization, and then provides a model that is consistent with the facts in that sector as well as with anecdotal accounts and other indicators of improved inventory management. The model is designed to address a number of concerns raised about inventory models in general, and in particular their ability to account convincingly for a number of facts about the Great Moderation.

A number of researchers (e.g. Blinder and Maccini, 1991; Humphries et al., 2001) have pointed out that inventory models, especially those applied to more aggregated data, have focused on finished goods inventories, whereas stocks of work-in-process and materials are actually larger and more volatile than those of finished goods. In addition, the durable goods sector in particular is regarded as primarily production-to-order rather than production-to-stock. So one contribution of this paper is to adapt the stockout-avoidance concept (as in Kahn, 1987) to a production-to-order setting, and to show that much of the intuition and results regarding production volatility still apply. In particular, the model shows that the need to make production-related decisions (in this case materials orders and work-in-process) in advance of information about final product demand results in production being more volatile than shipments. This is so in a model with no finished goods inventories.

A second contribution is to show that the model is capable of shedding light on the dramatic reduction in the volatility of output and related measures of real activity since the early 1980s. In this case the comparison will be with the change in the volatility of output and sales growth in the durable goods sector. We provide evidence that lead times for materials orders in manufacturing shrank after the early 1980s, and show that in the model this implies large reductions in the volatility of output growth, and more modest reductions in the volatility of sales growth.

1 The Reduced Volatility of the U.S. Economy: Revisiting the Facts

An abrupt drop in the volatility of U.S. real GDP growth in the early 1980s provided the initial impetus for research on The Great Moderation. Early findings of a discrete break in volatility around 1983 (McConnell and Perez-Quiros, 2000) encouraged a focus on comparisons before and after 1983. This approach conceals the fact that many economic series did not undergo an abrupt volatility drop around 1983. Some did so much earlier, some later. The “sudden drop” view also directs attention away from certain developments that perhaps play an important role in the long term decline of volatility. Structural shifts in the economy,

e.g., a rising share of services in aggregate output, are unlikely to produce an abrupt drop in aggregate volatility.

In this section we argue, echoing Blanchard and Simon (2001), that the suddenness of the volatility drop is more apparent than real—that, in fact, large shocks in the 1970s and a deep contraction in the early 1980s obscure longer term developments that contributed to a downward drift in volatility even before the 1980s. Figure 1 provides some evidence on this issue, showing quarterly annualized growth rates for the four NIPA sectors that comprise GDP: nondurable goods, durable goods, services, and structures. Each is scaled by its nominal share of GDP so that the magnitudes (in terms of growth contributions) are comparable, and the scale of the charts is the same as in Figure 1. It is clear from these figures that only in the durable goods sector did volatility change in much the same way—both in terms of magnitude and timing—as GDP. Nondurables output volatility dropped, but it had also been lower in the 1960s before increasing in the 1970s, and in any case it was never anywhere nearly as volatile as durables. Thus the decline—such as it was—is unlikely to have been a major factor in the stabilization of the early 1980s. Service sector output was also never nearly as volatile as durable goods output, and moreover, its volatility dropped substantially in the early 1960s, and again in the 1970s, long before the break in GDP volatility. Structures output did experience a drop in volatility at the same time as overall GDP, but the size of the sector and the magnitude of the contribution is modest.¹

That both in magnitude and timing, the drop in GDP volatility appears most closely related to developments in the durable goods sector is further illustrated in Figure 2. The top half plots rolling 5-year variances for the same four sectors and GDP, along with a “covariance term reflecting the variance of GDP not accounted for by the variances of the sectors. This figure also suggests that both for total GDP and especially for durables, the volatility decline in the early 1980s was an acceleration of a trend that dates back to World War II. Only the durables sector volatility exhibits a downward trend on the order of that followed by overall GDP volatility. The bottom half of Figure 2 shows the analogous chart for GDP broken down by expenditure categories. Prominent in the evolution of volatility is the inventory investment term and the covariance term.

Figure 2 is, in effect, just accounting, and does not prove cause and effect. It is possible that the decline in GDP volatility caused the decline in durables sector volatility, or in inventory investment volatility, or that all three had a common cause. Still, a challenge for any explanation of the overall decline in GDP volatility is to account for the specific patterns observed in this figure, as well as the more detailed facts regarding inventories and durable goods found below in Sections 2 and 3.

¹Note that the volatility contributions depicted in the charts are also affected by trends in sector shares over time, but the effect is very slight. The pictures would look virtually identical if sector shares were held constant.

As another illustration of the trend in volatility, we estimate a GARCH process for GDP growth, and for durables output growth, including time trends and other variables to explain changing volatility. The results are depicted in Table 2. The specification for the time series process is just an AR(1). The variance equation includes the usual GARCH terms, plus a time trend, and the trend squared (given that the variance has to remain positive). Also included in some specifications was the (lagged) 10-year treasury bond rate, to proxy for inflation and the volatility of the 1970s, and a dummy variable that takes on the value of one for the observations beginning in 1984Q1. The results show a significant downward trend in the variance, with the 10-year rate also coming in significantly, but with the post-1983 dummy not significant when added to the equation. Thus once one accounts for the volatility trend, and the uptick in volatility in the 1970s and early 1980s, it would appear that the post-1983 decline in volatility is better represented as the continuation of a longer-term trend than as a one-time break.

2 Changing Inventory Behavior

Since the early 1980s there have been a number of significant changes in the behavior of inventories in aggregate data.² Here we focus on the durable goods sector. While the inventory literature has traditionally focused on more disaggregated data, and in particular on the 2-digit (SIC) level manufacturing data, for the questions examined in this paper it is more appropriate to look at aggregate data. Disaggregated data can be misleading because it is impossible to tell whether changes in inventory behavior are genuine or just the result of a change in location or ownership of inventories. For example, if manufacturers decide not to hold finished goods inventories, but instead to ship goods to wholesalers or retailers immediately upon completion, that would appear as a big decline in manufacturing finished goods inventories, even though it would be largely offset by an increase in wholesale or retail inventories. Similarly, if manufacturers in one industry were to insist on “just-in-time” delivery of materials from suppliers, that could look like a dramatic decline in materials inventories for manufacturers, but it would likely be offset by increases in the inventories of suppliers, who would likely be from different industries.

A potentially important fact about the Great Moderation is that output volatility fell by substantially more than (and earlier than) final sales volatility, particularly in the durable goods sector.³ Since the difference between output and final sales is the change in inventories, this fact implies a change in inventory behavior—either a reduction in the volatility of inventory investment, or a change in the covariance between inventory investment and sales. Note that by convention, the service and structures sector do not carry inventories (in structures this is because final output includes construction in progress), so the source of

²See, for example, Kahn et al (2002).

³See McConnell and Perez-Quiros (2000), Kahn et al. (2002).

the change in inventory behavior must by definition lie in the goods sector.⁴ Figure 3 shows the behavior of output and sales volatility over time in the durable goods sector. In contrast to the behavior of output volatility, sales volatility shows only a modest decline.⁵

Given our focus on the volatility of real growth rates (as opposed to levels), we can examine a similar relationship between output, inventories and sales in terms of growth contributions. Although inventory investment, because it can be negative, does not have a conventionally defined growth contribution, we can define it indirectly as the difference between the growth rate of output and the growth contribution of final sales (cf. Kahn et al, 2002). Following Whelan (2000) we can approximate the latter in terms of the real growth rate of sales and the nominal share of sales in output. Letting γ_{xy} denote the growth contribution of x to output y , where $x = s$ for sales and $x = i$ for inventories, we define the growth contribution of inventory investment as

$$\gamma_{iy} = \gamma_{yy} - \gamma_{sy}$$

where $\gamma_{sy} = \gamma_{ss}\theta_{sy}$, θ_{sy} is the nominal share of s in y (measured as the average of current and lagged shares). The growth contribution of a variable to itself is just its real growth rate.

With these definitions in hand, we can track the contributions of sales and inventory investment to the variance of output growth over time:

$$\sigma_y^2 = \sigma_s^2 + \sigma_i^2 + 2\sigma_{si}$$

where the variances and covariance on the right-hand side refer to the growth contributions defined above. Figure 4 plots the three components for the durable goods sector. We see that both the inventory term and the covariance term exhibit a substantial downward trend, with the covariance term accounting in particular for the big drop in the early 1980s. Thus not only is the apparent break in 1984 associated with a change in inventory behavior, but the downward trend from the 1950s onward is as well.

In addition to this indirect evidence of changing inventory behavior, we can directly examine the inventory-sales ratio in the durable goods sector. Figure 5 shows that whether one looks at the ratio of real (in year 2000 dollars) inventories to real sales, or nominal to nominal, the ratio began a sharp decline in the early 1980s, at the same time that volatility in the sector declined. This is not by itself a proof of "progress;" it could just represent a shift along a fixed technological tradeoff in response to changing costs, or a compositional change within the sector. But the timing of the break in trend is striking.⁶

⁴There is, however, evidence of a change in inventory behavior in construction, even though it is not treated as such in the NIPAs. See Kahn (2000).

⁵McConnell and Perez-Quiros (2000) find evidence of a statistically significant break in the mid-1980s in durables output but not in final sales.

⁶While it is possible that inventory-holding has been pushed offshore—for example, the inventory data include materials stocks, and firms shift to foreign suppliers—it is worth noting that a similar time pattern is present in 2-digit manufacturing data excluding materials stocks from the inventory-sales ratio.

Secondly, the inventory-sales ratio is clearly less volatile (relative to its varying trend), suggesting that businesses either make smaller mistakes or are able to correct their inventories more quickly. Again this is not definitive as it could be that the shocks are smaller or that the industry composition has shifted. Kahn et al. (2002) also describes results from a VAR with sales and inventories that indicates a change in the variance decomposition pre- and post-1983. Before 1983 sales accounted for much more of the variance of inventories than inventories did of sales (37.8 percent versus 5.4 percent); after 1983 they were almost even (18.2 versus 14.9), consistent with the idea that firms were better able to anticipate sales and adjust inventories in advance. Moreover, the residual variance of sales dropped precipitously, meaning that less of the variation in sales was unpredicted given prior sales and inventories.

Although we have focused on the durable goods sector as a whole, for reasons emphasized earlier, we can examine disaggregated manufacturing data as well. This helps to alleviate concerns that the patterns in the aggregate sector are somehow misleading, either because they stem from compositional change (e.g. relative growth of less volatile industries within the sector) or are unrepresentative of a broad range of subsectors. Table 1 shows the volatility of production and sales growth for 2-digit durable goods manufacturing industries over the periods 1967-83 and 1984-1997. (The data are not available prior to 1967, and after 1997 the industry classifications were changed, so the series are not continuous.) The table shows a similar pattern across all eleven industries: a large reduction in the volatility of both output and sales growth. The only qualitative difference with the NIPA aggregate durables data is that for many industries the decline in sales volatility is approximately as large as the decline in output volatility. This suggests that some of the change in inventory behavior may occur more downstream in the wholesale or retail sectors. But the reduced volatility is clearly not due to compositional change, nor is it confined to a small subset of industries.

3 A Model of Durable Goods Production and Inventory Behavior

One approach to assessing the role of improved inventory control is to be agnostic about the details, but look for changes in parameters and propagation in, for example, a structural VAR. This is the approach in McCarthy and Zakrajsek (2007). They do find evidence of structural change pre- and post-1983. They use conventional identifying restrictions in an effort to sort out the role of, for example, monetary policy in altering the dynamics of the sales process.

A second approach is a more specific model of improved inventory control as in Kahn et al (2002), based on the approach in Kahn (1986) and Bils and Kahn (2000). Firms carry finished goods inventories to avoid stockouts in the face of uncertain demand, trading off the cost of foregone profits against the cost of carrying inventories. If demand is serially correlated, the mistakes will get magnified in production volatility, so that

it will exceed the volatility of sales. If technology enables firms to have better information about demand disturbances, then they will make smaller errors in their production decisions, and the additional volatility induced by correcting those errors is reduced. Firms may also be able to hold fewer inventories.

This type of mechanism can account for reduced production volatility (relative to the volatility of sales), but has several drawbacks. First, depending on the timing of the arrival of information, either the volatility of sales actually increases substantially, or the covariance of sales with inventory investment increases. As we have seen, the opposite is the case in the data. The reason sales volatility increases in this model is that the improved information essentially allows firms to accommodate demand shocks as opposed to damping them via stockouts. The covariance of sales and inventory investment only becomes more negative if the firm gets the information in time to adjust production sufficiently in advance (due to a desire to smooth production if costs are convex) that inventory movements anticipate the demand shock. Then when the shock occurs, inventory investment moves in the opposite direction, as anticipated by the firm. But this tends to exacerbate sales volatility.

The second problem with this approach, as alluded to in the introduction, is that it does not apply so obviously or directly to the durable goods sector, much of which is best characterized as production-to-order rather than production-to-stock. And as pointed out by Humphries et al (2001) and many others, most inventories, particularly in durable goods, are of materials or works in process, not final goods. Third, while there is much anecdotal evidence of technology that might provide better information about future sales, there is no direct evidence to assist in specifying a model. And as this discussion suggests, the details matter.

Because the model involves at least two types of stocks (works-in-process or “intermediate” goods, and unfilled orders of final goods), and will distinguish between materials orders and deliveries, a lot of notation is involved. Let D_t denote deliveries of materials at date t , which get combined with labor N_{Mt} to produce Y_{Mt} , which is the flow of intermediate goods that gets added to the stock M_t of works-in-process inventories at the end of period t . U_t is the stock of unfilled orders of final goods, X_t the flow from M_{t-1} into final production (i.e. gross output at the intermediate stage), Y_{Ft} , which corresponds to shipments of final goods, and O_t the flow of new orders for final goods. Value added at the final stage is $V_{Ft} = Y_{Ft} - X_{t-1} = A_{Ft}N_{Ft}$. At the intermediate stage, value added is $V_{Mt} = Y_{Mt} - D_{t-1} = A_M N_{Mt}$. We assume a Leontief technology for non-labor inputs at each stage. Thus we have

$$D_t = b_M Y_{Mt} \tag{1}$$

$$X_t = b_F Y_{Ft} \tag{2}$$

To simplify, we abstract from materials inventories and assume that all materials are immediately converted into works-in-process. Figure 7 provides a schematic diagram of the model.

A key element of the model is the delivery lag for materials orders, which we denote by τ . A longer lag means that when the firm makes a decision about materials orders it has less information about what the state of the economy will be when the materials arrive. Consequently the decisions will be less accurate, and will (as the model will show) induce greater volatility in production. By the same token, if firms are, by whatever means, whether it be information technology or management resources, able to shorten the lead time, they can reduce this source of volatility, and also potentially reduce average inventory holding costs.

While much has been written about information technology and inventory control, and in particular the push toward “just-in-time” inventory management, it is difficult to find direct and tangible evidence of improved inventory management. There is time series evidence on τ , however: the Institute for Supply Management (ISM) surveys manufacturers monthly on their lead time for materials orders, going back to 1955. It turns out that this lead time has varied substantially, and in particular has shortened since the early 1980s, at around the same time that output volatility declined. Figure 6 displays the time series. It is not ideal evidence, as it is not limited to durable goods producers. But it very clearly shows a lead time that is both distinctly lower and less volatile since the early 1980s.

There are undoubtedly other ways in which inventory management may have progressed during this time. In addition to shorter lead times enabling the firm to have better information about the state of the economy at the time of delivery, the firm may just have better information at any point in time. There is considerable anecdotal information about the use of information technology in inventory management to obtain improved information about product demand. For example, James Surowiecki, in the September 18, 2000 *New Yorker*, writes about the retailer Zara:

...Instead of reacting quickly to what customers want now, most retailers must guess what they’ll want six or nine months hence. That’s hard enough if you’re selling televisions or bicycles. In the fashion business, it’s close to impossible.

Zara doesn’t have to worry about any of that.... It does not overstock, and unsuccessful designs are often whisked off shelves after just a week, so the company doesn’t have to slash prices. Equipped with handheld devices linked directly to the company’s design rooms in Spain, Zara’s store managers can report daily on what customers are buying, scorning, and asking for but not finding. Most important, the company takes just ten to fifteen days to go from designing a product—which, to be sure, often means knocking off a hot new look—to selling it.

This idea of better information at any point in time was incorporated into the Kahn et al (2002) model for a production-to-stock technology. But while it is undoubtedly part of the larger story of improved inventory management, it is difficult to quantify. The primary reason for focusing on shorter lead times is not because it is the only, or even the most important, aspect of improved inventory management. It is just that there is some quantitative evidence on it, however limited.

3.1 One-Period Lead Time

Given the discrete time nature of the model, for the sake of simplicity we will assume that τ is an integer, representing the number of periods ahead (“lead time”) the firm must order materials before they will arrive. To start with we will assume a one-period lead time for materials. So the timing is as follows: An materials order Z_t results in a delivery D_{t+1} and intermediate production $Y_{M,t+1} = b_M^{-1}D_{t+1}$, with final production and shipment $Y_{F,t+1}$ at $t + 1$. Materials costs are incurred upon delivery.

Assuming a constant wage w and a price q of materials, the cost of producing Y_M is $c_M Y_M = (w/A_M + qb_M) Y_M$, and total the cost of producing $Y_{F,t}$ is $c Y_{F,t}$, where

$$c = (w/A_F + c_M b_F) = w/A_F + (w/A_M + qb_M) b_F$$

The firm incurs additional costs from carrying inventories of works-in-process.

We assume that prices are fixed and final goods orders O_t follow a stochastic process, which for concreteness we assume is a simple AR(1):

$$O_t = \rho O_{t-1} + \eta_t \tag{3}$$

We also assume that there is no “spec” final production, so X_t is chosen only to fill known (i.e. unfilled) orders as of the beginning of period t , which do not include new orders O_t . The idea is that final production involves customization that can only be done for a specific order. Intermediate production is more generic, and can be done speculatively. X_t may also be constrained by the availability of works-in-process $M_{t-1} + b_M^{-1}D_t$.

We assume the firm maximizes profits subject the various technological constraints:

$$E_0 \left\{ \sum_{\tau=t}^{\infty} \beta^{\tau-t} [pY_{F\tau} - qD_{\tau} - w(N_{M\tau} + N_{F\tau})] \right\}$$

subject to

$$M_t = M_{t-1} + Y_{Mt} - X_t \quad (4)$$

$$U_t = U_{t-1} + O_t - Y_{Ft} \quad (5)$$

$$M_t \geq 0 \quad (6)$$

$$U_t \geq O_t. \quad (7)$$

$$V_{it} = A_i N_{it}, \quad i = M, F \quad (8)$$

$$Z_{t-1} = D_t \quad (9)$$

$$M_{t-1} + Y_{Mt} \geq X_t \quad (10)$$

$$U_{t-1} \geq Y_{Ft} \quad (11)$$

where V_i denotes value added, Y_i gross output at each stage ($i = M, F$), and $\beta < 1$ is a discount factor. For p sufficiently large (that is, for a positive markup), the firm will always try to fill all unfilled orders U_{t-1} at date t . This implies

$$X_t = \min \{M_{t-1} + b_M^{-1} D_t, b_F U_t\} \quad (12)$$

$$U_{t+1} = U_t + O_{t+1} - b_F^{-1} X_t \quad (13)$$

$$= O_{t+1} + \max \{0, U_t - b_F^{-1} (M_{t-1} + D_t)\} \quad (14)$$

That is, subject to availability of works-in-process, X_t is chosen to fill all unfilled orders U_t .

Note the timing assumptions here: All new orders in period t are unfilled as of the end of t ; shipments during t are for orders placed at $t-1$ or earlier. Whether or not new orders O_t are filled by the end of period $t+1$ depends on the adequacy of materials orders Z_t , which were made before O_t was known. Also note that under certainty, say with constant final goods orders \bar{O} , the above setup implies that $X = Y_M = b_F \bar{O}$, $Y_F = \bar{O}$, $Z = D = b_M b_F \bar{O}$, $N_M = b_F \bar{O} / A_M$, $N_F = \bar{O} / A_F$, and $M_t = 0$, $U_t = \bar{O} \dots$ There is no reason to hold inventories if both deliveries and orders are known in advance. Unfilled orders are held only because of the assumption that “spec” production is infeasible due to customization requirements.

We can show that the optimal ordering rule decision rule is of the form

$$Z_t = b_M [b_F E_t \{U_{t+1}\} + \kappa_1 - M_t] = D_{t+1} \quad (15)$$

where κ_1 is a constant to be determined (see below). It is then straightforward to show that

$$U_{t+1} = \eta_t + O_{t+1} - \min \{b_F^{-1}\kappa_1, \eta_t\} \quad (16)$$

$$X_t = b_F (\eta_{t-1} + E_{t-1} \{O_t\}) - \min \{\kappa_1, b_F \eta_{t-1}\} + \min \{\kappa_1, b_F \eta_t\} \quad (17)$$

$$M_t = \max \{\kappa_1 - b_F \eta_t, 0\} \quad (18)$$

$$D_t = b_M b_F (\eta_{t-1} + E_{t-1} \{O_t\}) \quad (19)$$

$$Z_t = b_M b_F (\eta_t + E_t \{O_{t+1}\}) \quad (20)$$

Whether or not the constraint (10) is binding is reflected in terms like $\min \{\kappa_1, b_F \eta_{t-1}\}$ and $\max \{\kappa_1 - b_F \eta_t, 0\}$. But note that Z_t is not affected by past constraints, nor does it end up depending on κ_1 . If $\kappa_1 < b_F \eta_t$, X_t is constrained by materials. This should add to $E_t \{U_{t+1}\}$, which it does. But the effect of η_t on Z_t occurs regardless of the outcome of $\min \{\kappa_1, b_F \eta_t\}$, because if there is no stockout (so no increase in unfilled orders) there is the same impact on M_t , which also adds to Z_t . In other words, as η_t increases, either M_t falls or $E_t \{U_{t+1}\}$ rises, and both have the same impact on Z_t . X_t is affected by the outcome of $\min \{\kappa_1, b_F \eta_t\}$ because higher η_t can mean that X_t is constrained by the stock of works-in-process. Note that if $\kappa_1 < b_F \eta_t$, there will be unfilled orders carried over into $t + 1$ (i.e. $U_{t+1} > O_{t+1}$), as the firm had insufficient works-in-process to fill U_t , so some of those unfilled orders get carried over into $t + 1$. If $\kappa_1 > b_F \eta_t$ then $U_{t+1} = O_{t+1}$.

In the National Income and Product Accounts (NIPA), “sales” are really total expenditures on final goods. This corresponds to shipments Y_{Ft} , i.e.

$$Y_{Ft} = b_F^{-1} X_t = \eta_{t-1} + E_{t-1} \{O_t\} - \min \{b_F^{-1}\kappa_1, \eta_{t-1}\} + \min \{b_F^{-1}\kappa_1, \eta_t\}. \quad (21)$$

“Production” Y_t is shipments plus the change in inventories, i.e.

$$\begin{aligned} Y_t &= \eta_{t-1} + E_{t-1} \{O_t\} - \min \{b_F^{-1}\kappa_1, \eta_{t-1}\} + \\ &\quad \min \{b_F^{-1}\kappa_1, \eta_t\} + b_F \max \{b_F^{-1}\kappa_1 - \eta_t, 0\} - b_F \max \{b_F^{-1}\kappa_1 - \eta_{t-1}, 0\} \end{aligned} \quad (22)$$

It is easy to see that production can be more volatile than sales, in particular when the O_t process exhibits positive serial correlation. Letting $\nu_t \equiv \min \{b_F^{-1}\kappa_1, \eta_t\}$, we can simplify the above expressions (using the

fact that $\max\{b_F^{-1}\kappa_1 - \eta_t, 0\} = b_F^{-1}\kappa_1 - v_t$:

$$\begin{aligned} Y_{Ft} &= \eta_{t-1} + E_{t-1}\{O_t\} + \Delta\nu_t \\ Y_t &= \eta_{t-1} + E_{t-1}\{O_t\} + (1 - b_F)\Delta\nu_t \end{aligned}$$

Note that $\Delta\nu_t$ is negatively correlated with $\eta_{t-1} + E_{t-1}\{O_t\}$ (at least for $\rho \geq 0$), which helps to explain why the variance of production can exceed the variance of sales: Inventory investment covaries positively with shipments. The focus in this paper, however, is not on this, but on how production and shipments volatility vary with τ .

Finally, what is κ_1 ? It is straightforward to show that it follows from the first-order condition:

$$\beta p \Pr(b_F U_{t+1} > M_t + D_{t+1}/b_M) - \beta c + \quad (23)$$

$$\beta^2 c [1 - \Pr(b_F U_{t+1} > M_t + D_{t+1}/b_M)] = 0. \quad (24)$$

where c is as defined earlier, the total unit cost of producing the final good. Given

$$U_{t+1} = \eta_t + O_{t+1} - v_t \quad (25)$$

we have

$$U_{t+1} = E_t\{U_{t+1}\} + \eta_{t+1}. \quad (26)$$

The probability is of a materials stockout at date $t + 1$, and also represents $dE_t(X_{t+1})/dZ_t$, the expected impact on “completions” at $t + 1$ from an additional order of materials at t . The intuition is that by ordering an additional unit at date t at cost βc , with some probability the firm gains an additional sale at date $t + 1$ (the event of a work-in-process stockout at date $t + 1$), and with one minus that probability it results in surplus stocks and an offsetting decrease in materials orders at date $t + 1$.

Suppose η_t has a c.d.f. of G . We then get

$$\Pr(b_F [E_t\{U_{t+1}\} + \eta_{t+1}] > M_t + Z_t/b_M) = \frac{c(1 - \beta)}{p - \beta c} \quad (27)$$

which implies

$$\kappa_1 = b_F G^{-1}\left(\frac{p - c}{p - \beta c}\right). \quad (28)$$

which, as one would expect, is increasing in the markup and decreasing in the discount rate $1/\beta - 1$.

3.2 Shorter or Longer Delivery Lags

3.2.1 Zero Lead Time

Now consider one extreme: No delivery lag, so $D_t = Z_t$. With a little algebra, we can show that

$$X_t = b_F O_t \quad (29)$$

$$M_t = 0 \quad (30)$$

$$Z_t = D_t = b_M b_F O_t \quad (31)$$

$$Y_{Ft} = O_{t-1} = Y_t \quad (32)$$

Clearly going from $\tau = 1$ to $\tau = 0$ changes the relationship between production and sales volatility. As we shall see, for the realistic case in which O_t exhibits positive serial correlation, shrinking the delivery lag from one to zero obviously reduces the gap between the variance of production and the variance of sales. This case is relatively trivial, but nonetheless it provides some insight into why the more interesting case we consider next, shrinking the delivery lag from two periods to one, has the same qualitative impact of reducing the volatility of both production and shipments, but with the former declining more than the latter.

3.2.2 Two-Period Lead Time

We now consider a delivery lag of two periods. We hypothesize a decision rule

$$D_t = Z_{t-2} = b_M [b_F E_{t-2} \{U_t\} + \kappa_2 - E_{t-2} \{M_{t-1}\}] \quad (33)$$

where, again, $\kappa_2 \neq \kappa_1$ is to be determined. We then have

$$X_t = \min \{M_{t-1} + b_F E_{t-2} \{U_t\} + \kappa_2 - E_{t-2} \{M_{t-1}\}, b_F U_t\} \quad (34)$$

$$U_{t+1} = O_{t+1} + \max \{U_t - E_{t-2} \{U_t\} - b_F^{-1} \kappa_2 - b_F^{-1} [M_{t-1} - E_{t-2} \{M_{t-1}\}], 0\} \quad (35)$$

$$M_t = \max \{\kappa_2 - [b_F (U_t - E_{t-2} \{U_t\}) - (M_{t-1} - E_{t-2} \{M_{t-1}\})], 0\} \quad (36)$$

Let $\xi_t \equiv \eta_t + (1 + \rho) \eta_{t-1}$ and $u_t = \min \{b_F^{-1} \kappa_2, \xi_t\}$. With considerably more algebra than in the $\tau = 1$ case, we can show (see Appendix) that

$$M_t = b_F \max \{b_F^{-1} \kappa_2 - \xi_t, 0\} \quad (37)$$

$$Y_{Ft} = E_{t-2} \{O_t\} + \Delta u_t + (1 + \rho) \eta_{t-2} \quad (38)$$

Determining κ_2 turns out to be a more complex problem than in the $\tau = 1$ case. It is straightforward to show that it follows from the first-order condition

$$\beta^2 p \Pr(b_F U_{t+2} > M_{t+1} + D_{t+2}/b_M) - \beta^2 c + \beta^4 c [1 - \Pr(b_F U_{t+2} > M_{t+1} + D_{t+2}/b_M)] = 0. \quad (39)$$

The probability is now of a works-in-process stockout at date $t + 2$, and also represents $dE_t(X_{t+2})/dZ_t$. The intuition is that by ordering an additional unit at date t at cost $\beta^2 c$, with some probability the firm gains an additional sale at date $t + 2$ (the event of a works-in-process stockout), and with one minus that probability it results in surplus stocks and an offsetting decrease in materials orders at date $t + 2$. Suppose ξ_t has a c.d.f. of F . We then get

$$\Pr\left(\xi_{t+2} > \frac{D_{t+2}}{b_M b_F} - [E_t\{U_{t+2}\} - b_F^{-1} E_t\{M_{t+1}\}]\right) = \frac{c(1 - \beta^2)}{p - \beta^2 c} \quad (40)$$

$$G\left(\frac{D_{t+2}}{b_M b_F} - [E_t\{U_{t+2}\} - b_F^{-1} E_t\{M_{t+1}\}]\right) = \frac{p - c}{p - \beta^2 c} \quad (41)$$

$$\frac{D_{t+2}}{b_M b_F} = F^{-1}\left(\frac{p - c}{p - \beta^2 c}\right) + E_t\{U_{t+2}\} - b_F^{-1} E_t\{M_{t+1}\} \quad (42)$$

$$Z_t = b_M [\kappa_2 + b_F E_t\{U_{t+2}\} - E_t\{M_{t+1}\}] \quad (43)$$

where

$$\kappa_2 = b_F F^{-1}\left(\frac{p - c}{p - \beta^2 c}\right). \quad (44)$$

Any difference between κ_1 and κ_2 would stem from the difference in the relevant distribution function (F vs. G) and from the fact that excess orders are more costly because they take two periods to offset. Of course, either of these differences could affect the firm's markup.

Finally, what is $b_F E_t\{U_{t+2}\} - E_t\{M_{t+1}\}$? We have

$$b_F E_t\{U_{t+2}\} - E_t\{M_{t+1}\} = b_F E_t [O_{t+2} + \max\{\xi_{t+1} - b_F^{-1} \kappa_2, 0\} - \max\{b_F^{-1} \kappa_2 - \xi_{t+1}, 0\}] \quad (45)$$

$$= b_F E_t\{O_{t+2}\} + E_t\{\xi_{t+1} - b_F^{-1} \kappa_2\} \quad (46)$$

$$= b_F E_t\{O_{t+2}\} + b_F (1 + \rho) \eta_t - \kappa_2 \quad (47)$$

So

$$Z_t = b_M b_F [E_t\{O_{t+2}\} + (1 + \rho) \eta_t] \quad (48)$$

If we compare (48) and (20) we see that now in addition to having to order based on two-period-ahead expected orders, the date t innovation has a magnified impact (to the extent $\rho > 0$). Table 3 gives a

comparison of the major variables of interest for the two cases.

Regarding implications for inventory-sales ratios, it turns out that the model is incomplete: Absent a theory of the markup p/c in the two cases ($\tau = 1$ and $\tau = 2$), the relative size of inventory-sales ratios is ambiguous. But under the reasonable assumption that $(p - c)/(p - \beta c)$ for $\tau = 1$ and $(p - c)/(p - \beta^2 c)$ for $\tau = 2$ are the same (implying that p/c is larger for $\tau = 2$), and that markups are larger than the discount rate $1/\beta - 1$, then inventory-sales ratios will be larger under $\tau = 2$, essentially because of the greater uncertainty at the time of ordering.

3.3 Aggregation

The solutions for the time series behavior output, shipments, and inventories are not realistic characterizations of any data that are likely to be observed. In practice, we observe aggregates, even when we look at relatively disaggregated data. The data are typically aggregates of different goods, different locations, and different firms. Consequently we never see zeros of any stocks, whether of unfilled orders or inventories.

Fortunately the model is amenable to aggregation as follows. We now suppose a continuum of symmetric firms, each of which faces stochastic orders as above, but with an idiosyncratic shock v_{it} . That is, for firm i ,

$$O_{it} = \rho O_{t-1} + \eta_t + v_{it}. \quad (49)$$

where for concreteness we can assume that v_{it} is normally distributed with mean zero and variance σ_v^2 , and that $\int_{-\infty}^{\infty} v\phi(v) = 0$. Of course, the derivations of κ_1 and κ_2 must now be revised to reflect idiosyncratic risk. For example, G in (28) should be the distribution function for $\eta_t + v_{it}$.

Let

$$\begin{aligned} \zeta_{1t} &= \int_{-\infty}^{\infty} \min\{\kappa_1, \eta_t + \sigma_v v\} \phi(v) dv \\ &= \int_{-\infty}^{\frac{\kappa_1 - \eta_t}{\sigma_v}} (\eta_t + \sigma_v v) \phi(v) dv + \kappa_1 \left[1 - \Phi\left(\frac{\kappa_1 - \eta_t}{\sigma_v}\right) \right] \end{aligned} \quad (50)$$

The idea here is that each firm's outcome varies depending on its idiosyncratic shock, so we integrate to get the aggregate. But because of the linear technology, the firms adjust and go into the next period looking identical. So for aggregate inventories we have (in the $\tau = 1$ case)

$$M_t = b_F (\kappa_1 - \zeta_{1t}) \quad (51)$$

Similarly,

$$Y_{Ft} = \eta_{t-1} + \rho O_{t-1} + \zeta_{1t} - \zeta_{1t-1} \quad (52)$$

$$Y_{Ft} + \Delta M_t = \eta_{t-1} + \rho O_{t-1} + (1 - b_F)(\zeta_{1t} - \zeta_{1t-1})$$

We can linearize ζ_{1t} around $\eta_t = 0$:

$$\zeta_{1t} \approx \kappa_1 \left[1 - \Phi \left(\frac{\kappa_1}{\sigma_v} \right) \right] + \Phi \left(\frac{\kappa_1}{\sigma_v} \right) \eta_t = \theta_1 \eta_t \quad (53)$$

where

$$\theta_1 \equiv \Phi \left(\frac{\kappa_1}{\sigma_v} \right) \quad (54)$$

and we ignore the constant term, which is not relevant for the exercise of computing volatility. Note that for finite κ_1 , $\theta \in (0, 1)$. So we have

$$Y_{Ft} = (1 + \rho - \theta) \eta_{t-1} + \rho^2 O_{t-2} + \theta \eta_t \quad (55)$$

$$Y_t = Y_{Ft} + \Delta M_t = (1 + \rho - \theta(1 - b_F)) \eta_{t-1} + \rho^2 O_{t-2} + \theta(1 - b_F) \eta_t$$

$$\text{Var}(Y_{Ft}) \approx (1 + \rho - \theta)^2 \sigma_\eta^2 + \rho^4 \sigma_\eta^2 / (1 - \rho^2) + \theta^2 \sigma_\eta^2$$

$$\text{Var}(Y_t) \approx (1 + \rho - \theta(1 - b_F))^2 \sigma_\eta^2 + \rho^4 \sigma_\eta^2 / (1 - \rho^2) + \theta^2 (1 - b_F)^2 \sigma_\eta^2$$

So

$$[\text{Var}(Y_t) - \text{Var}(Y_{Ft})] / \sigma_\eta^2 = (1 + \rho - \theta(1 - b_F))^2 - (1 + \rho - \theta)^2 + \theta^2 (1 - b_F)^2 - \theta^2 \quad (56)$$

$$= 2\theta b_F (\rho - 2\theta + \theta b_F + 1) \quad (57)$$

$$> 0 \quad \text{if } 1 + \rho > (2 - b_F)\theta \quad (58)$$

So for sufficiently large ρ or b_F , or small θ (that is, small κ_1), production is more volatile than sales. Note that ρ need not even be positive for this to be true.

With $\tau = 2$ we define (assuming for simplicity that idiosyncratic risk is i.i.d.):

$$\zeta_{2t} = \int_{-\infty}^{\infty} \min\{\kappa_2, \xi_t + \sigma_v v\} \phi(v) dv \quad (59)$$

$$= \int_{-\infty}^{\frac{\kappa_2 - \xi_t}{\sigma_v}} (\xi_t + \sigma_v v) \phi(v) dv + \kappa_2 \left[1 - \Phi\left(\frac{\kappa_2 - \xi_t}{\sigma_v}\right) \right] \quad (60)$$

$$\approx \Phi\left(\frac{\kappa_2}{\sigma_v}\right) \xi_t = \theta_2 \xi_t \quad (61)$$

We then get

$$M_t = b_F (\kappa_2 - \zeta_{2t}) \quad (62)$$

and

$$Y_{Ft} = E_{t-2}\{O_t\} + (1 + \rho)\eta_{t-2} + \zeta_{2t} - \zeta_{2t-1} \quad (63)$$

$$= \rho^2 O_{t-2} + \theta_2 (\eta_t + \rho\eta_{t-1}) + (1 - \theta_2)(1 + \rho)\eta_{t-2} \quad (64)$$

$$Y_t = E_{t-2}\{O_t\} + (1 + \rho)\eta_{t-2} + (1 - b_F)(\zeta_{2t} - \zeta_{2t-1}) \quad (65)$$

$$= \rho^2 O_{t-2} + (1 - b_F)\theta_2 [\eta_t + \rho\eta_{t-1}] + (1 - (1 - b_F)\theta_2)(1 + \rho)\eta_{t-2}$$

So

$$\text{Var}(Y_F) / \sigma_\eta^2 = \rho^6 / (1 - \rho^2) + \theta_2^2 (1 + \rho^2) + [(1 - \theta_2)(1 + \rho) + \rho^2]^2 \quad (66)$$

$$\text{Var}(Y) / \sigma_\eta^2 = \rho^6 / (1 - \rho^2) + (1 - b_F)^2 \theta_2^2 (1 + \rho^2) + [(1 - (1 - b_F)\theta_2)(1 + \rho) + \rho^2]^2 \quad (67)$$

and

$$[\text{Var}(Y) - \text{Var}(Y_F)] / \sigma_\eta^2 = 2\theta_2 b_F (\rho^2 + \rho + 1) (\rho - 2\theta_2 + \theta_2 b_F + 1) \quad (68)$$

Now compare $2\theta b_F (\rho - 2\theta + \theta b_F + 1)$ versus $2\theta_2 b_F (\rho^2 + \rho + 1) (\rho - 2\theta_2 + \theta_2 b_F + 1)$. Clearly if the first is positive, the second is larger for $\rho > 0$. So holding fixed the shock variance σ_η^2 , for parameters in the empirically relevant range, a reduction in the delivery lag (i.e. the lead time for materials orders) results in a reduction in both output and sales volatility, but a greater reduction in output volatility.

3.4 Simulations

We can get some feel for the capability of this approach to account for changes in volatility. We choose parameters that roughly match the relevant characteristics of the data in the early part of the sample (1954-1983) under the assumption that $\tau = 2$, and then simulate a change to $\tau = 1$. This is not to suggest that

there actually was such an abrupt change, it is just the limitation of the discrete-time nature of the model.

The simulations involved the following parameters: $\rho = 0.95$, $\sigma_\eta^2 = 0.1$, $\sigma_\nu^2 = 1$, $b_F = 0.75$, and $(p - c)/(p - \beta c)$ for $\tau = 1$ and $(p - c)/(p - \beta^2 c)$ for $\tau = 2$ are both set equal to 0.95, which corresponds to about a 20% markup if $\beta = 0.99$. (The markup p/c is assumed to be larger in the $\tau = 2$ case to make up for the larger inventory holding costs). These parameters imply an “annualized” standard deviation of the growth rate of orders of about 6.5%, and imply $\kappa_1 = 1.38$, $\kappa_2 = 1.87$, $\theta_1 = 0.92$, $\theta_2 = 0.97$.

The results of the simulation are shown in Table 4. The level effects are modest (as they are in the data, though moreso), but the growth rate effects are large. The standard deviation of η was chosen so that the simulation would match the pre-1984 standard deviation of output for $\tau = 2$ (see the last line of Table 1). The model qualitatively matches the basic facts about reduced volatility in the durable goods sector: Initially production volatility exceeds sales volatility by a lot. Subsequently, both volatilities go down, but production volatility declines by much more than sales volatility. Both volatility declines in the model are smaller than in the data, however.

It should be emphasized that the model is far too stylized for the simulation to be taken seriously as a calibration exercise. Nonetheless, the fact that it at least qualitatively matches the facts suggests that it may be a reasonable basis for pursuing a more realistic extension, perhaps along the lines that Bills and Kahn (2000) extended Kahn (1987) for empirical purposes.

4 Conclusions

[to be added]

Appendix

This appendix provides a derivation of the result for $\tau = 2$ that materials orders take the form (33)

$$D_t = Z_{t-2} = b_M [b_F E_{t-2} \{U_t\} + \kappa_2 - E_{t-2} \{M_{t-1}\}],$$

and that, consequently,

$$\begin{aligned} M_t &= b_F \max \{b_F^{-1} \kappa_2 - \xi_t, 0\} \\ Y_{Ft} &= E_{t-2} \{O_t\} + \Delta u_t + (1 + \rho) \eta_{t-2} \end{aligned}$$

The strategy is to assume the form, derive the implications for the evolution of the endogenous variables, and then show that (33) satisfies the first-order condition. Let

$$\begin{aligned} x_t &\equiv b_F (U_t - E_{t-2} \{U_t\}) \\ z_t &\equiv M_t - E_{t-1} \{M_t\} \end{aligned}$$

Then

$$\begin{aligned} U_t &= O_t + b_F^{-1} \max \{x_{t-1} - z_{t-2} - \kappa_2, 0\} \\ M_t &= \max \{\kappa_2 - (x_t - z_{t-1}), 0\}. \end{aligned}$$

Therefore

$$\begin{aligned} U_t - b_F^{-1} M_{t-1} &= O_t + b_F^{-1} (x_{t-1} - z_{t-2} - \kappa_2) \\ E_{t-2} \{U_t - b_F^{-1} M_{t-1}\} &= E_{t-2} \{O_t + b_F^{-1} (x_{t-1} - z_{t-2} - \kappa_2)\} \\ U_t - b_F^{-1} M_{t-1} - E_{t-2} \{U_t - b_F^{-1} M_{t-1}\} &= O_t - E_{t-2} \{O_t\} + b_F^{-1} (x_{t-1} - E_{t-2} \{x_{t-1}\}) \\ &= O_t - E_{t-2} \{O_t\} + (U_{t-1} - E_{t-3} \{U_{t-1}\}) - \\ &\quad E_{t-2} \{(U_{t-1} - E_{t-3} \{U_{t-1}\})\} \\ &= O_t - E_{t-2} \{O_t\} + U_{t-1} - E_{t-2} \{U_{t-1}\} \\ U_{t-1} - b_F^{-1} M_{t-2} - E_{t-3} \{U_{t-1} - b_F^{-1} M_{t-2}\} &= O_{t-1} - E_{t-3} \{O_{t-1}\} + U_{t-2} - E_{t-3} \{U_{t-2}\} \end{aligned}$$

So

$$\begin{aligned}
M_t &= \max \{ \kappa_2 - [b_F (U_t - E_{t-2} \{U_t\}) - (M_{t-1} - E_{t-2} \{M_{t-1}\})], 0 \} \\
&= \max \{ \kappa_2 - b_F [O_t - E_{t-2} \{O_t\} + U_{t-1} - E_{t-2} \{U_{t-1}\}], 0 \} \\
U_t &= O_t + b_F^{-1} \max \{ b_F (U_{t-1} - E_{t-3} \{U_{t-1}\}) - (M_{t-2} - E_{t-3} \{M_{t-2}\}) - \kappa_2, 0 \} \\
&= O_t + \max \{ U_{t-1} - E_{t-3} \{U_{t-1}\} - b_F^{-1} (M_{t-2} - E_{t-3} \{M_{t-2}\}) - b_F^{-1} \kappa_2, 0 \} \\
&= O_t + \max \{ O_{t-1} - E_{t-3} \{O_{t-1}\} + U_{t-2} - E_{t-3} \{U_{t-2}\} - b_F^{-1} \kappa_2, 0 \} \\
\\
M_t &= b_F \max \{ b_F^{-1} \kappa_2 - [O_t - E_{t-2} \{O_t\} + U_{t-1} - E_{t-2} \{U_{t-1}\}], 0 \} \\
U_t &= O_t + \max \{ O_{t-1} - E_{t-3} \{O_{t-1}\} + U_{t-2} - E_{t-3} \{U_{t-2}\} - b_F^{-1} \kappa_2, 0 \} \\
\\
U_{t-1} &= O_{t-1} + \max \{ O_{t-2} - E_{t-4} \{O_{t-2}\} + U_{t-3} - E_{t-4} \{U_{t-3}\} - b_F^{-1} \kappa_2, 0 \} \\
E_{t-2} \{U_{t-1}\} &= E_{t-2} \{O_{t-1}\} + E_{t-2} \{ \max \{ O_{t-2} - E_{t-4} \{O_{t-2}\} + U_{t-3} - E_{t-4} \{U_{t-3}\} - b_F^{-1} \kappa_2, 0 \} \} \\
U_{t-1} - E_{t-2} \{U_{t-1}\} &= O_{t-1} - E_{t-2} \{O_{t-1}\} \\
M_t &= b_F \max \{ b_F^{-1} \kappa_2 - [O_t - E_{t-2} \{O_t\} + O_{t-1} - E_{t-2} \{O_{t-1}\}], 0 \} \\
&= b_F \max \{ b_F^{-1} \kappa_2 - [\eta_t + (1 + \rho) \eta_{t-1}], 0 \} \\
U_t &= O_t + \max \{ O_{t-1} - E_{t-3} \{O_{t-1}\} + O_{t-2} - E_{t-3} \{O_{t-2}\} - b_F^{-1} \kappa_2, 0 \} \\
&= O_t + \max \{ \eta_{t-1} + (1 + \rho) \eta_{t-2} - b_F^{-1} \kappa_2, 0 \} \\
\\
D_t &= b_M [b_F E_{t-2} \{U_t\} + \kappa_2 - E_{t-2} \{M_{t-1}\}] \\
D_t/b_M &= b_F E_{t-2} \{O_t\} + E_{t-2} \{ \max \{ b_F [\eta_{t-1} + (1 + \rho) \eta_{t-2}] - \kappa_2, 0 \} \} \\
&\quad - E_{t-2} \{ \max \{ \kappa_2 - b_F [\eta_{t-1} + (1 + \rho) \eta_{t-2}], 0 \} \} + \kappa_2 \\
&= b_F E_{t-2} \{O_t + (1 + \rho) \eta_{t-2}\}
\end{aligned}$$

Finally

$$\begin{aligned}
Y_{Ft} &= b_F^{-1} X_t \\
&= b_F^{-1} \min \{M_{t-1} + b_F E_{t-2} \{U_t\} + \kappa_2 - E_{t-2} \{M_{t-1}\}, b_F U_t\} \\
&= U_t - \max \{(U_t - E_{t-2} \{U_t\}) - b_F^{-1} (M_{t-1} - E_{t-2} \{M_{t-1}\}) - b_F^{-1} \kappa_2, 0\} \\
&= U_t - \max \{O_t - E_{t-2} \{O_t\} + U_{t-1} - E_{t-2} \{U_{t-1}\} - b_F^{-1} \kappa_2, 0\} \\
&= O_t + \max \{[\eta_{t-1} + (1 + \rho) \eta_{t-2}] - b_F^{-1} \kappa_2, 0\} \\
&\quad - \max \{[\eta_t + (1 + \rho) \eta_{t-1}] - b_F^{-1} \kappa_2, 0\} \\
&= O_t + \max \{\xi_{t-1} - b_F^{-1} \kappa_2, 0\} - \max \{\xi_t - b_F^{-1} \kappa_2, 0\} \\
&= O_t + \max \{\xi_{t-1} - b_F^{-1} \kappa_2, 0\} - \max \{\xi_t - b_F^{-1} \kappa_2, 0\} \\
&= O_t + (\xi_{t-1} - u_{t-1}) - (\xi_t - u_t) \\
&= E_{t-2} \{O_t\} + \eta_t + \rho \eta_{t-1} + \Delta u_t - \Delta \xi_t \\
&= E_{t-2} \{O_t\} + \Delta u_t + (1 + \rho) \eta_{t-2}
\end{aligned}$$

And since

$$\begin{aligned}
\max \{\xi_{t-1} - b_F^{-1} \kappa_2, 0\} &= \xi_{t-1} - \min \{b_F^{-1} \kappa_2, \xi_{t-1}\} = \xi_{t-1} - u_{t-1} \\
\max \{\kappa_2 - b_F \xi_t, 0\} &= b_F \max \{b_F^{-1} \kappa_2 - \xi_t, 0\} = b_F (b_F^{-1} \kappa_2 - u_t)
\end{aligned}$$

we have

$$\begin{aligned}
U_t - E_{t-2} \{U_t\} - b_F^{-1} [M_{t-1} - E_{t-2} \{M_{t-1}\}] &= O_t - E_{t-2} \{O_t\} + \max \{\xi_{t-1} - b_F^{-1} \kappa_2, 0\} \\
&\quad - E_{t-2} \{\max [\xi_{t-1} - b_F^{-1} \kappa_2, 0]\} \\
&\quad - [\max \{b_F^{-1} \kappa_2 - \xi_{t-1}, 0\} - E_{t-2} \{\max \{b_F^{-1} \kappa_2 - \xi_{t-1}, 0\}\}]
\end{aligned}$$

or

$$\begin{aligned}
U_t - E_{t-2} \{U_t\} - b_F^{-1} [M_{t-1} - E_{t-2} \{M_{t-1}\}] &= \eta_t + \rho \eta_{t-1} + \xi_{t-1} - b_F^{-1} \kappa_2 - E_{t-2} \{\xi_{t-1} - b_F^{-1} \kappa_2\} \\
&= \eta_t + \rho \eta_{t-1} + \eta_{t-1} + (1 + \rho) \eta_{t-2} - (1 + \rho) \eta_{t-2} \\
&= \xi_t
\end{aligned}$$

which we can substitute into the first-order condition (39) to show that (33) is in fact a solution to the

optimization problem.

References

- [1] Bils, Mark, and Kahn, James A. “What Inventory Behavior Tells Us about Business Cycles.” *American Economic Review*, June 2000, pp. 458-481.
- [2] Blanchard, Olivier, and John Simon. “The Long and Large Decline in U.S. Output Volatility,” *Brookings Papers on Economic Activity* 1, 2001, pp. 135 – 164.
- [3] Blinder, Alan, and L. Maccini, “Taking Stock: A Critical Assessment of Recent Research on Inventories,” *Journal of Economic Perspectives*, 5 (Winter 1991), 73-96.
- [4] Comin, Diego, and Sunil Mulani, “Diverging Trends in Macro and Micro Volatility: Facts,” manuscript, 2003, New York University.
- [5] Davis, Steven, and James Kahn, “Changes in the Volatility of Economic Activity at the Macro and Micro Levels,” manuscript, December 2007.
- [6] Kahn, James A., “Explaining the Gap between New Home Sales and Inventories,” Federal Reserve Bank of New York, *Current Issues in Economics and Finance*, 6 (May 2000).
- [7] Kahn, James A. “Inventories and the Volatility of Production,” *American Economic Review* 77 (September 1987), pp. 667-679.
- [8] Kahn, James, Margaret M. McConnell, and Gabriel Perez-Quiros. “On the Causes of the Increased Stability of the U.S. Economy,” *Federal Reserve Bank of New York Economic Policy Review* 8 (2002), 183 – 202.
- [9] Humphreys, B. , L. Maccini , S. Schuh, “Input and Output Inventories,” *Journal of Monetary Economics* 47 (2001), 347-375.
- [10] Maccini, Louis, and Adrian Pagan. “Exploring the Role of Inventories in the Business Cycle,” manuscript, 2003.
- [11] McCarthy, Jonathan, and Egon Zakrajsek, “Inventory Dynamics and Business Cycles: What Has Changed?” *Journal of Money, Credit and Banking* 39, (March–April 2007), 591-613.
- [12] McConnell, Margaret M, and Gabriel Perez-Quiros. “Output Fluctuations in the United States: What has Changed Since the Early 1980’s,” *American Economic Review* 90(5), 2000, pp. 1464 – 1476

- [13] Ramey, Valerie A., and Daniel J. Vine. "Tracking the Source of the Decline in GDP Volatility: An Analysis of the Automobile Industry," manuscript, 2003.
- [14] Stock, James H., and Mark W. Watson. "Has the Business Cycle Changed and Why?" *NBER Macroeconomics Annual*, 2002, pp. 159 – 218.
- [15] Stock, James H., and Mark W. Watson. "Has the Business Cycle Changed? Evidence and Explanations," prepared for the Federal Reserve Bank of Kansas City symposium, "Monetary Policy and Uncertainty," Jackson Hole, Wyoming, August 28-30, 2003.

Table 1: Durable Goods Manufacturing Volatility

Industry (SIC code)	S.D. of output growth		S.D. of sales growth	
	1967-83	1984-97	1967-83	1984-97
Lumber (24)	23.15	17.18	20.36	13.70
Furniture/Fixtures (25)	15.38	8.77	14.57	7.41
Stone, Clay, Glass (32)	14.60	10.30	13.93	9.89
Primary Metals (33)	27.88	9.96	27.78	8.94
Fabricated Metals (34)	16.35	8.59	14.29	7.14
Industrial Machinery (35)	14.63	9.76	12.24	8.97
Electronic Machinery (36)	14.94	7.64	13.57	7.47
Motor Vehicles (371)	46.04	22.98	45.59	21.59
Other Transportation (37x)	23.37	17.09	17.14	16.19
Instruments (38)	11.42	7.22	10.84	5.79
Miscellaneous (39)	20.79	11.51	18.62	10.25
Durable Manufacturing	14.66	7.09	13.72	6.39
Durable Goods (aggregate)*	18.32	7.81	11.04	8.26

*Early sample is 1954-83, late sample is 1984-2007Q3

Table 2: GARCH results

Dependent variable						
	GDP growth			Durables output growth		
Const	2.250	2.249	2.313	4.581	4.697	5.010
	(0.272)	(0.310)	(0.296)	(0.742)	(0.800)	(0.832)
Lagged Dep Var.	0.317	0.309	0.303	0.087	0.101	0.086
	(0.070)	(0.074)	(0.074)	(0.067)	(0.071)	(0.074)
Variance equation						
Const	6.515	11.118	12.911	251.83	303.24	182.08
	(0.781)	(2.139)	(6.190)	(6.901)	(44.57)	(68.66)
Resid ²	0.096	0.057	0.125	0.035	0.027	0.062
	(0.058)	(0.063)	(0.090)	(0.041)	(0.044)	(0.058)
GARCH(-1)	0.713	0.658	0.334	0.566	0.477	0.468
	(0.089)	(0.149)	(0.388)	(0.079)	(0.149)	(0.249)
Trend	-0.045	-0.121	-0.068	-1.788	-3.239	-1.415
	(0.006)	(0.012)	(0.003)	(0.207)	(0.149)	(0.068)
Trend ²	8.03E-05	0.0003	1.97E-04	0.0033	0.0079	4.65E-04
	3.95E-05	0.0001	(4.11E-06)	(0.0010)	(0.0015)	5.50E-05
10-year T-rate(-1)	—	0.294	0.210	—	9.317	8.457
		(0.202)	(0.181)		(4.008)	(3.171)
Dummy(≥ 1984.1)	—	—	-7.001	—	—	-116.19
			(5.192)			(66.05)

Note: Estimated on the sample 1947Q1-2007Q3

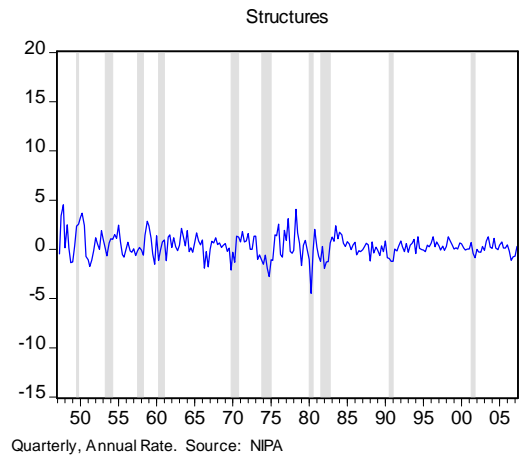
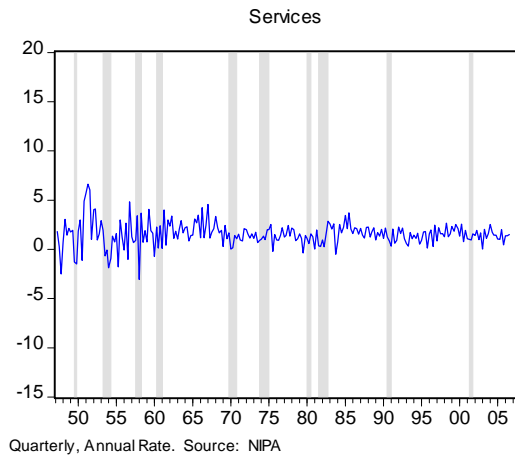
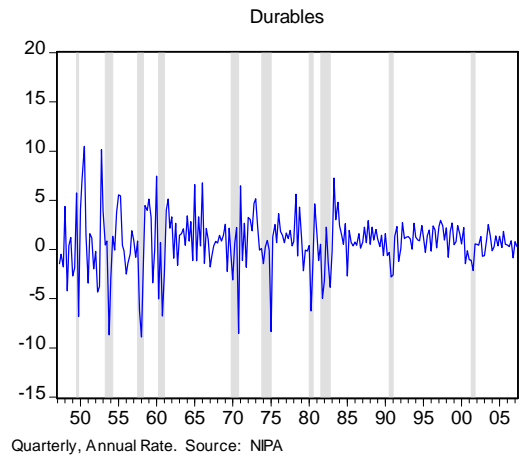
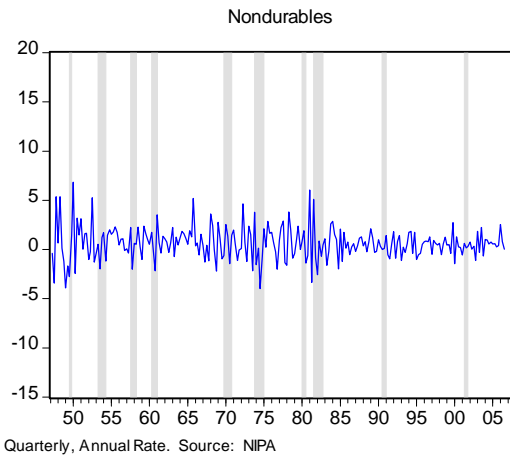
Table 3: Summary of Results for Different Lead Times τ

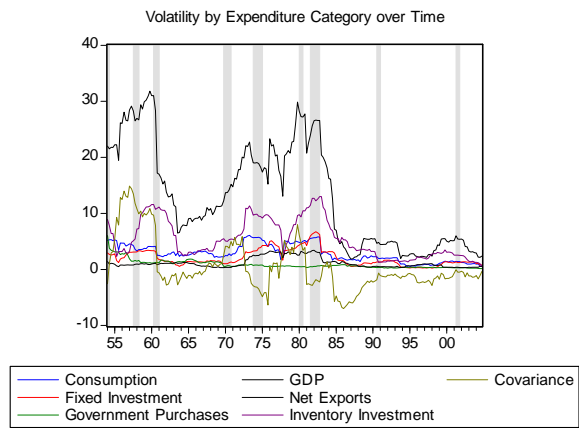
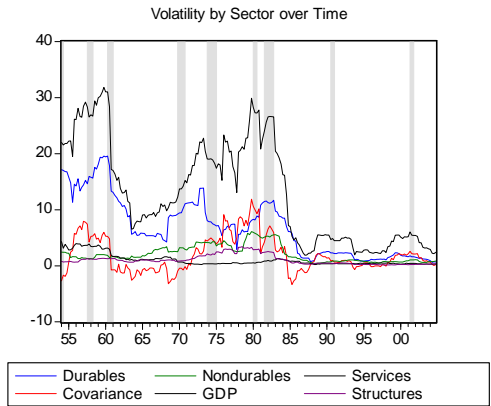
	$\tau = 0$	$\tau = 1$	$\tau = 2$
D_t/b_M	$b_F O_t$	$b_F [E_{t-1} \{O_t\} + \eta_{t-1}]$	$b_F [E_{t-2} \{O_t\} + (1 + \rho) \eta_{t-2}]$
U_t	O_t	$O_t + \eta_{t-1} - \nu_{t-1}$	$O_t + \xi_{t-1} - u_{t-1}$
M_t	0	$\kappa_1 - b_F \nu_t$	$\kappa_2 - b_F u_t$
Y_{Ft}	O_{t-1}	$E_{t-1} \{O_t\} + \eta_{t-1} + \Delta \nu_t$	$E_{t-2} \{O_t\} + (1 + \rho) \eta_{t-2} + \Delta u_t$
Y_t	O_{t-1}	$E_{t-1} \{O_t\} + \eta_{t-1} + (1 - b_F) \Delta \nu_t$	$E_{t-2} \{O_t\} + (1 + \rho) \eta_{t-2} + (1 - b_F) \Delta u_t$

Table 4: Simulation Results

Standard deviations of				
	Output	Sales	Output growth	Sales Growth
$\tau = 2$	2.35	2.27	18.00	6.45
$\tau = 1$	2.30	2.27	11.46	6.18

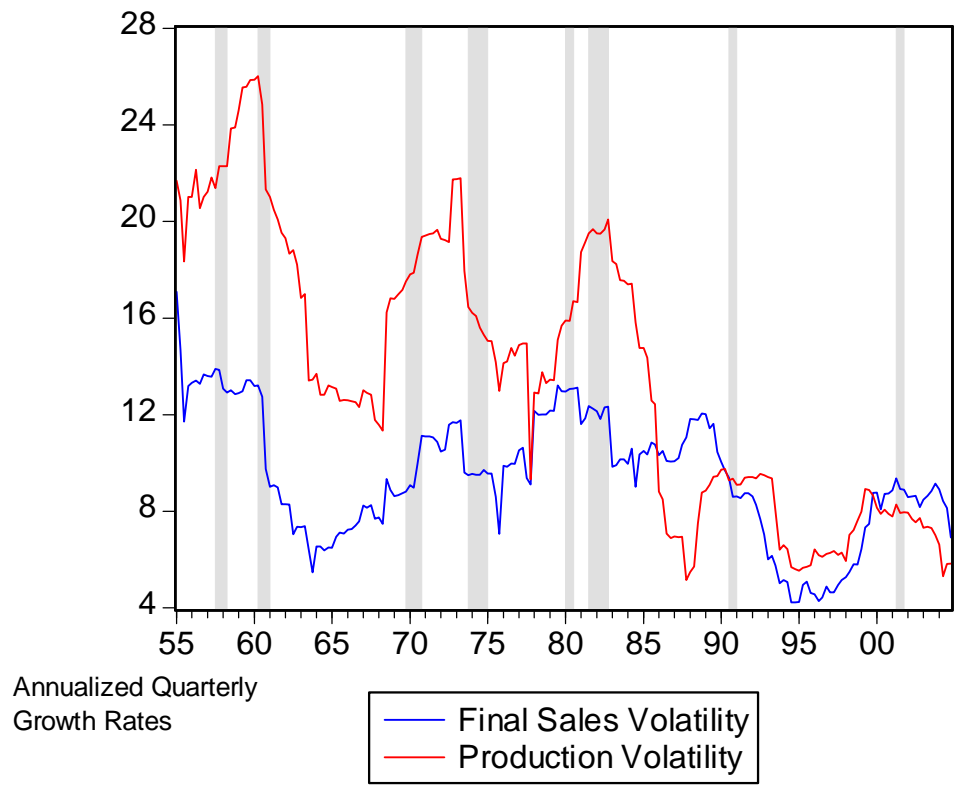
Comparison of Share-Weighted Sectoral Growth Rates





5-year rolling variances, scaled by sector's share

Durable Goods Sector Volatility
(5-Year Centered Rolling Standard Deviations)



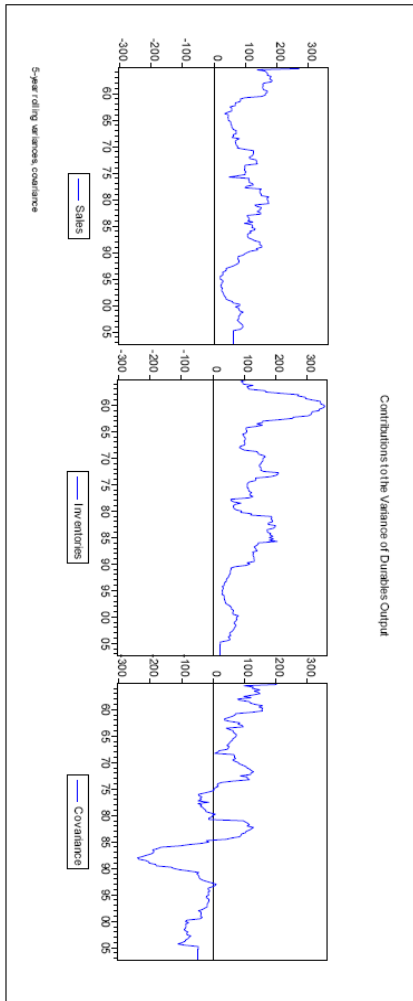
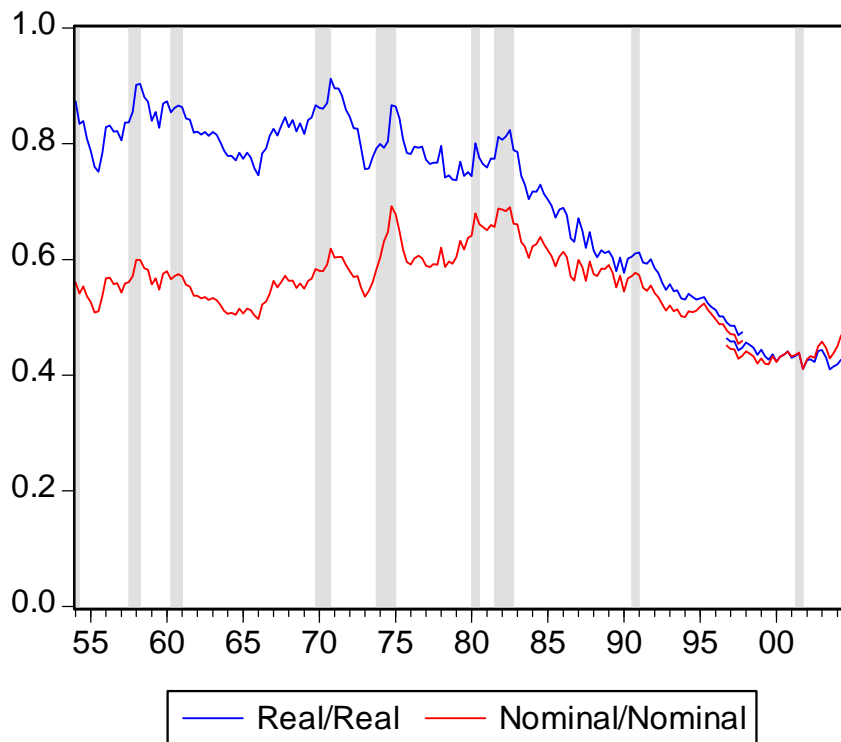


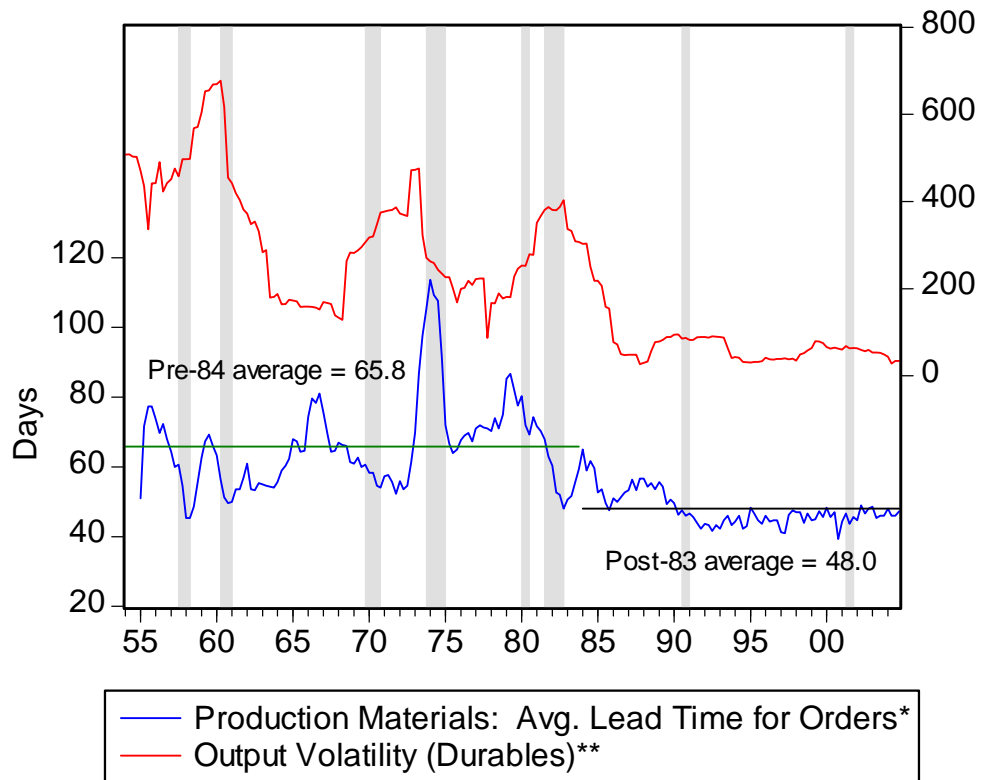
Figure 4

Durable Goods Inventory-Sales Ratio*



*The series is discontinuous (and overlapping for one year) due to the change from SIC to NAICS industry definitions

Durables Volatility and Lead Time for Orders



* Source: ISM survey

**5-year centered rolling variance

Production to Order

